ICT 103: Electrical Science

UNIT 1 : DC Circuits

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Module 1

D.C. Circuits

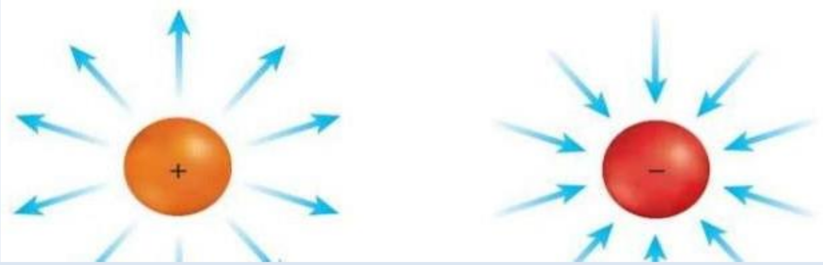
Electricity: Electricity plays an important role in our day to day life.
Electricity is used for

1. Lighting (lamps)
2. Heating(heaters)
3. Cooling
4. Entertainment (T.V. and radio)
5. Transportation
6. Calculations(Calculators)

Now- a- days all the activities are dependent upon electricity.
Electricity: The invisible energy which constitutes flow of electrons in a closed circuit to do work is called electricity.

Nature of Electricity: Every matter is electrical in nature since it contains charged particles like electrons and protons. Therefore

1. Ordinarily, a body is neutral as it contains same number of protons and electrons.
2. If some of electrons are removed from the body, there is a deficit of electrons and the body attains a positive charge.
3. If some of electrons are supplied to the body, there occurs excess of electrons and the body attains a negative charge.



A body is said to be charged +vely or –vely if it has deficit or excess of electrons from its normal due share respectively.

Unit of Charge:
The practical unit of charge is coulomb.
One Coulomb= charge on 6.28 x 10^{18} electrons.

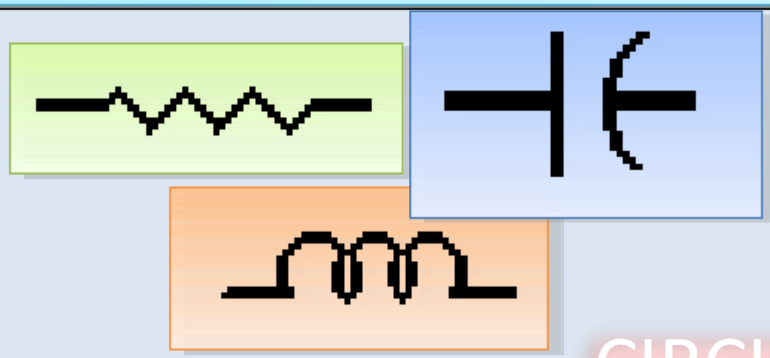
CIRCUITS

CIRCUITS

- A closed conducting path through which an electric current flows or is intended to flow

Parameters

- The various elements of an electric circuit, like resistance, inductance, and capacitance which may be lumped or distibuted.



Free Electrons: The valence electrons which are loosely attached to the nucleus of an atom and free to move when external energy is applied are called free electrons.

Electrical Potential: The capacity of charged body to do work is called electrical potential.

$$\textbf{Electrical Potential} = \frac{\textbf{Workdone}}{\textbf{Charge}} = \frac{\textbf{W}}{\textbf{Q}}$$

$$V = \frac{\textbf{W}}{\textbf{Q}}$$

Unit of electrical potential is **Vollts or Joules/Coulomb.**

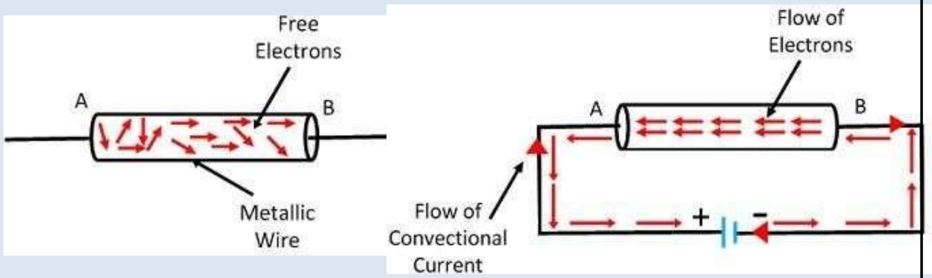
Def: A body is said to have an electric potential of 1 Volt if 1 Joule of work is done to charge the body to 1 coulomb.

Potential Difference:

The difference in electrical potential of the two charged bodies is called potential difference.
Unit of potential difference is **Volts.**

Electric Current: In metallic wire, a large number of electrons are available which move from one atom to other at random.

When an electrical potential is applied across a metallic wire, the loosely attached free electron start moving towards positive terminal of the cell.



Thus, continuous flow of electrons in an electric circuit is called electric current

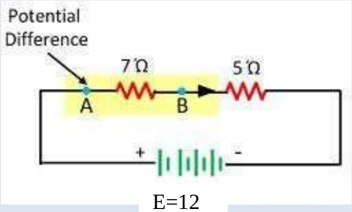
Definition-

Current is rate of flow of electrons i.e. charge flowing per second.

$$\textbf{I} = \frac{Q}{t}$$

The unit of current is Ampere (A)

E.M.F. (Electromotive force) and potential difference:
E.m.f is the force that causes an electric current to flow in an electric circuit.
Infact it is not a force but it is an energy.
E.m.f: **The electromotive force is the amount of energy supplied by the source to each coulomb of charge.**
Potential Difference: **The potential difference is the amount of energy used by the one coulomb of charge in moving from one point to the other.**
In the following figure battery has emf of 12V and the potential difference between A and B is 7V.



E=12

Ohm's Law

Ohm"s laws state that the current through any two points of the conductor is directly proportional to the potential difference applied across the conductor, provided physical conditions i.e. temperature, etc. do not change. It is measured in (Ω) ohm.

Mathematically it is expressed as

$$I \propto V$$

$$\frac{V}{I} = constant$$

$$\frac{V_1}{I_1} = \frac{V_2}{I_2} = \cdots = \frac{V_n}{I_n} = constant$$

This constant is also called the resistance (R) of the conductor (or circuit)

R= $\frac{V}{I}$

In a circuit, when current flows through a resistor, the potential difference across the resistor is known as voltage drops across it, i.e., V = IR.

Limitations of Ohm’s Law

- Ohm‟s law is not applicable in unilateral networks. Unilateral networks allow the current to flow in one direction. Such types of network consist of elements like a diode, transistor, etc.
- It is not applicable for the non-linear network (network containing non-linear elements such as electric arc etc). In the nonlinear network, the parameter of the network is varied with the voltage and current. Their parameter likes resistance, inductance, capacitance and frequency, etc., not remain constant with the times. So ohms law is not applicable to the nonlinear network. Ohm‟s law is used for finding the resistance of the circuit and also for knowing the voltage and current of the circuit.

Resistance: The opposition offered to flow of current is called resistance. It is represented by R. The unit of resistance is ohms (Ω)

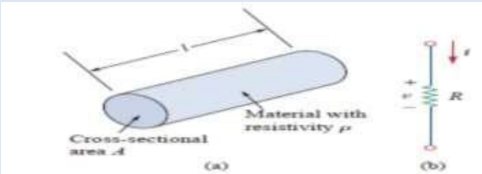


Fig. 1.13 (a) Typical Resistor, (b) Circuit Symbol for Resistor

Law of Resistance: The resistance of a wire depends upon

1. It is directly proportional to its length.
   R α L ………………. 1
2. It is inversely proportional to its area of cross-section.
   R α $\frac{1}{A}$ ………………..2
3. It depends upon the nature of material of which the wire is made.
4. It also depends upon the temperature of the wire.
   Combining 1 and 2
   R α $\frac{L}{A}$
   R = $\frac{\rho L}{A}$

The proportionality constant ρ is called the specific resistance o resistivity of the conductor and its value depends on the material of which the conductor is made.

The inverse of the resistance is called the conductance and inverse of resistivity is called specific conductance or conductivity. The symbol used to represent the conductance is G and conductivity is σ. Thus conductivity $\sigma = 1/\rho$ and its units are Siemens per meter

$$G = \frac{1}{R} = \frac{A}{\rho l} = \frac{1}{\rho} \cdot \frac{A}{l} = \sigma \cdot \frac{A}{l}$$

Numerical on OHM's law

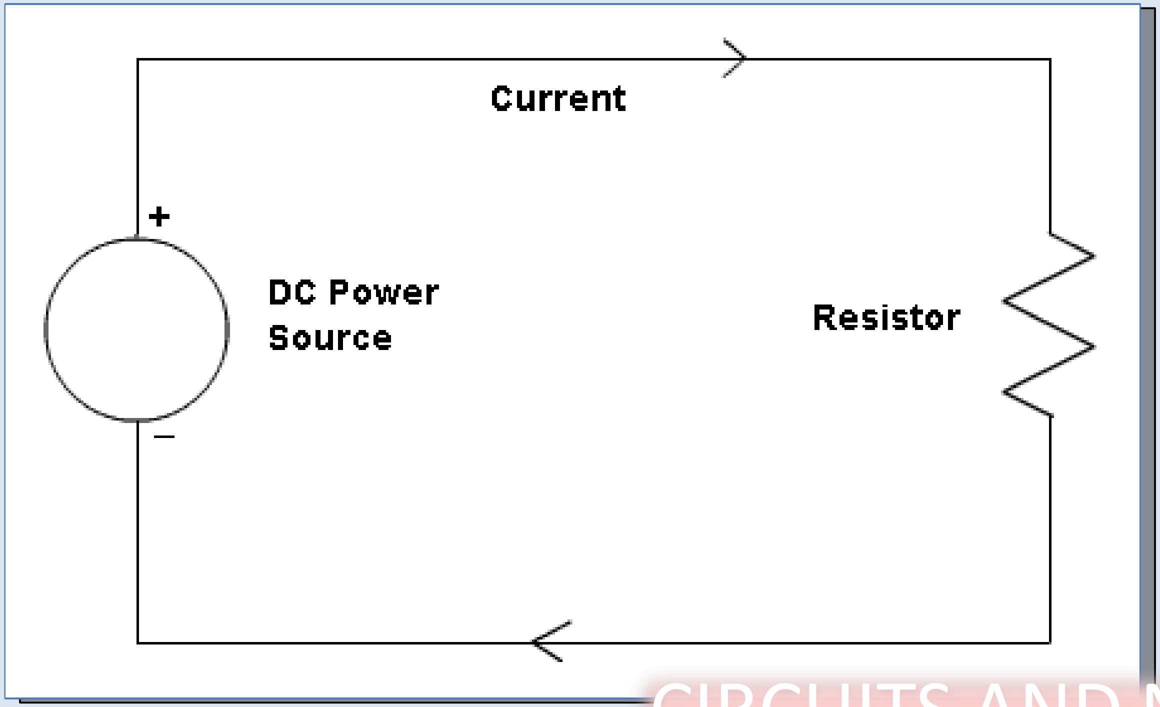
1. A wire of length 1 m has a resistance of 2 ohms. Obtain the resistance if specific resistance is double, the diameter is double and the length is made three times the first.

2. There are two wires A& B of same material. A is 20 times longer than B and has one fifth of the cross-section as that of B. If the resistance of A is 1 ohm. What is the resistance of B?

OHM'S LAW

OHM'S LAW

- One of the most fundamental law in electrical circuits relating voltage, current and resistance
- Developed in 1827 by German physicist **Georg Simon Ohm**



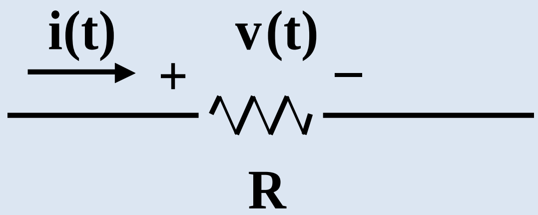
OHM'S LAW

- According to Ohm's Law, the current (I) flowing in an electrical circuit is directly is directly proportional to the applied voltage (E) and inversely proportional to the equivalent resistance (R) of the circuit and mathematically expressed as:

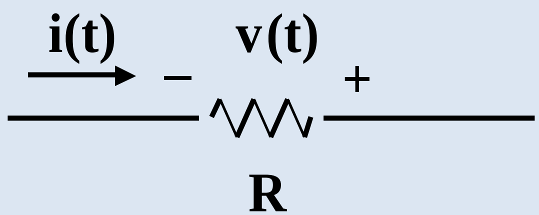
$$I = \frac{E}{R}$$

Ohm's Law:

The current flowing through the electric circuit is directly proportional to the potential difference across the circuit and inversely proportional to the resistance of the circuit, provided the temperature remains constant.



$$v(t) = Ri(t) \qquad (2.1)$$

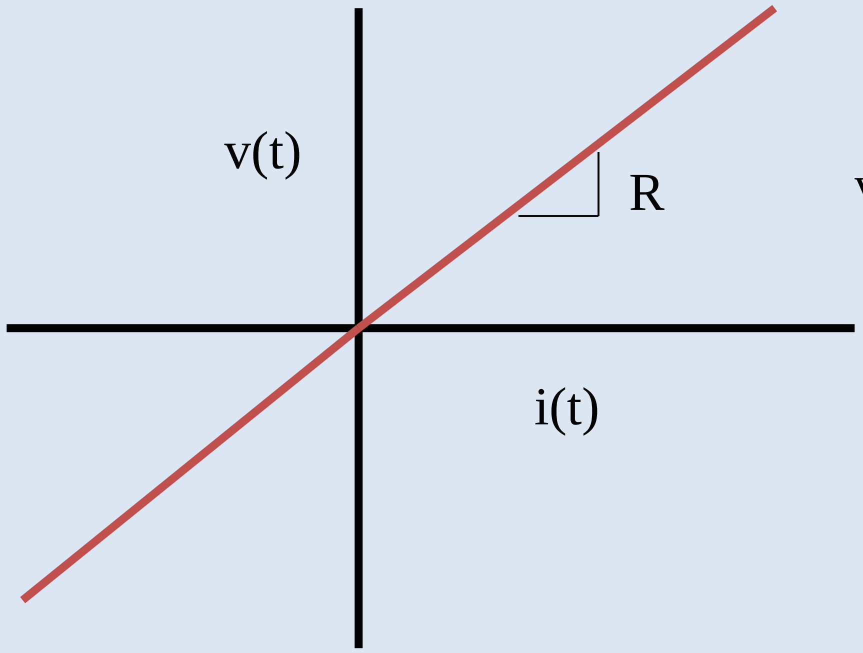


$$v(t) = -Ri(t) \qquad (2.2)$$

Basic Laws of Circuits

Ohm's Law:

Directly proportional means a straight line relationship.



v(t) = Ri(t)

The resistor is a model and will not produce a straight line for all conditions of operation.

Basic Laws of Circuits

Ohm's Law: About Resistors:

The unit of resistance is ohms(Ω).

A mathematical expression for resistance is

$$R = \rho \frac{l}{A} \qquad (2.3)$$

l : *The length of the conductor (meters)*

A : *The cross-sectional area* $(meters^2)$

ρ : *The resistivity* $(\Omega \cdot m)$

Basic Laws of Circuits

Ohm's Law: About Resistors:

We remember that resistance has units of ohms. The reciprocal of resistance is conductance. At one time, conductance commonly had units of mhos (resistance spelled backwards).

In recent years the units of conductance has been established as siemens (S).

Thus, we express the relationship between conductance and resistance as

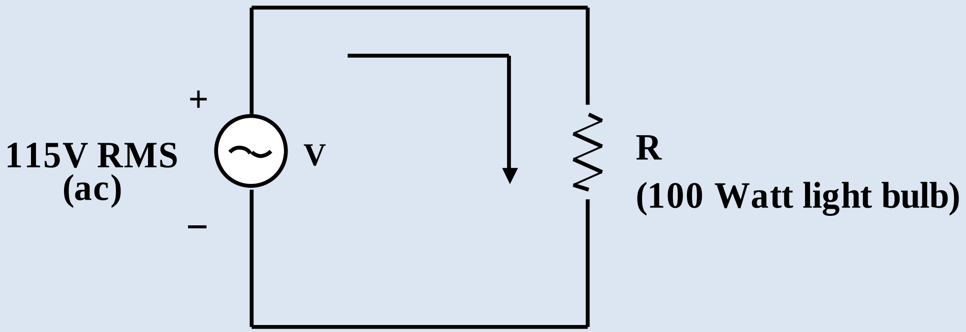
$$G = \frac{1}{R} \quad \text{(S)} \qquad (2.4)$$

We will see later than when resistors are in parallel, it is convenient to use Equation (2.4) to calculate the equivalent resistance.

Basic Laws of Circuits

Ohm's Law: Ohm's Law: Example 2.1.

Consider the following circuit.



Determine the resistance of the 100 Watt bulb.

$$P = VI = \frac{V^2}{R} = I^2 R \qquad (2.5)$$

$$R = \frac{V^2}{P} = \frac{115^2}{100} = 132.25 \; ohms$$

A suggested assignment is to measure the resistance of a 100 watt light bulb with an ohmmeter. Debate the two answers.

CIRCUITS

TYPES

Linear Circuit

- Is one whose parameters are constant (i.e. They do not change with voltage and current.

Non-Linear Circuit

- Is that circuit whose parameters change with voltage and current.

Bilateral Circuit

- Is one whose properties or characteristics are the same in either direction.

Unilateral Circuit

- Is that circuit whose properties or characteristics change with the direction of its operation.

ELECTRICAL NETWORKS

ELECTRICAL NETWORK

- Connection of various electric elements in any manner

TYPES

<u>Passive Network</u>

- ➢With no source of emf.

<u>Active Network</u>

- ➢Contains one or more than one sources of emf.

DEFINATIONS

Linear elements :
In an electric circuit, a linear element is an electrical element with a linear relationship between current and voltage. Resistors are the most common example of a linear element; other examples include capacitors, inductors, and transformers.

Nonlinear Elements :
A nonlinear element is one which does not have a linear input/output relation. In a diode, for example, the current is a non-linear function of the voltage.Most semiconductor devices have non-linear characteristics.

Active Elements :
The elements which generates or produces electrical energy are called active elements. Some of the examples are batteries, generators,transistors,operational amplifiers,vacuum tubes etc.

Passive Elements :
All elements which consume rather than produce energy are called passive elements, like resistors,inductors and capacitors.

In unilateral element, voltage – current relation is not same for both the direction. Example: Diode, Transistors.

In bilateral element, voltage – current relation is same for both the direction. Example: Resistor

The voltage generated by the source does not vary with any circuit quantity. It is only a function of time. Such a source is called an ideal voltage Source.

The current generated by the source does not vary with any circuit quantity. It is only a function of time. Such a source is called as an ideal current source.

Resistance : It is the property of a substance which opposes the flow of current through it. The resistance of element is denoted by the symbol "R". It is measured in Ohms. $R = PL / A \ \Omega$

Electric Circuit:

The close path for flow of electric current is called electric circuit. The electric circuit is an arrangement of electrical energy sources and various circuit elements such as R, L and C are connected in series, parallel or series parallel combinations.

Circuit Elements:

The circuit elements can be categorized as:

1. Active and passive elements
2. Unilateral and bilateral elements
3. Linear and non-linear elements
4. Lumped and distributed elements

1. Active and passive elements:

Active elements are those who supply energy or power in the form of a voltage or current to the circuit or network.

Examples of the active components are batteries or generators etc.

Passive elements are those who receive energy in the form of voltage or current.

Examples of the passive components are resistor, capacitor and inductor.

2. Unilateral and bilateral elements:

Unilateral elements: The elements which conduct the current in one direction only are called unilateral elements such as diodes, transistors, vacuum tubes, rectifiers etc

Bilateral elements: The elements which conduct the current in both the directions are called bilateral elements such as resistors.

3. Linear and non-linear elements

Linear Elements: The elements which follow the linear relation between current and voltage.

e.g. resistors

Non Linear Elements: The elements which don‟t follow the linear relation between current and voltage. e.g. Diode and transistors

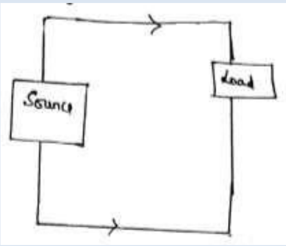
4. Lumped and distributed elements:

Lumped elements: The elements in which action takes place simultaneously are lumped elements such as resistor, capacitor and inductor. These elements are smaller in size.

Distributed elements: The elements in which for a given cause is not occurring simultaneously at the same instant but it is distributed are called distributed elements such as transmission lines.

Voltage and Current Source:

To deliver electrical energy to the electrical circuits, a source is required and a load is connected to source as shown in fig.



The source may be d.c. source or a.c source.

D.C. source:

Any source that produces direct voltage continuously and has ability to deliver direct current is called d.c. source such as batteries and generators etc.

A.C. source:

Any source that produces alternating voltage continuously and has ability to deliver the alternating current is called a.c. source such as alternators, oscillators or signal generators.

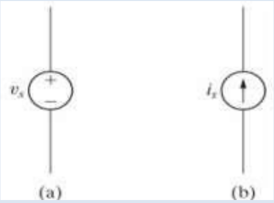
Independent and dependent sources:

There are two types of sources- Voltage source and current source. Sources can be either independent or dependent upon some other quantities.

Independent voltage/ current source:

The voltage (a.c or d.c.) does not dependent on other voltages or current in the circuit.

Symbol for independent voltage and current source



Examples of independent voltage source batteries and generators.
Examples of independent current source semiconductor devices such as Diode and transistors

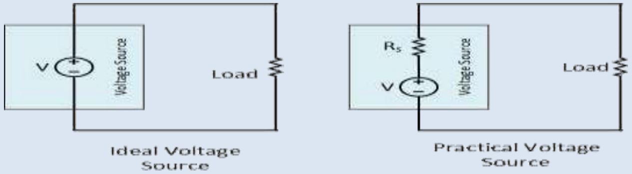
Dependent voltage/ current source:

The voltage does dependent on another voltage or current in the circuit.
Symbol for dependent voltage and current source

(a) Dependent voltage source (b) Dependent current source

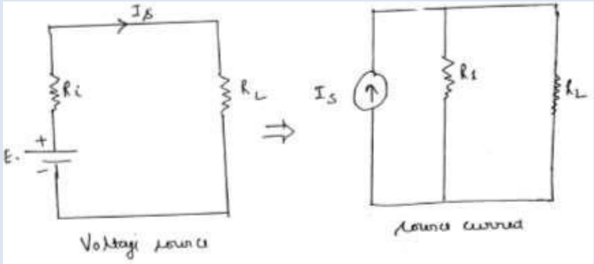
Ideal and practical voltage sources:

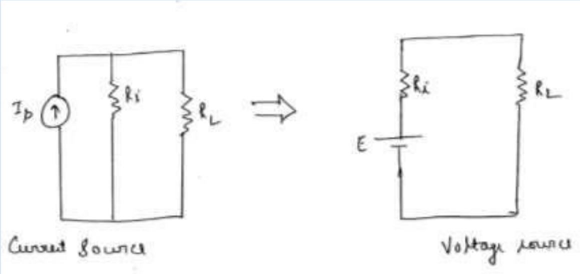
Ideal voltage sources: An imaginary voltage source, which can provide a constant voltage to load ranging from zero to infinity. Such voltage source is having zero internal resistance, Rs and is called Ideal Voltage Source. Practically it is not possible to build a voltage source with no internal resistance and constant voltage for that long range of the load.

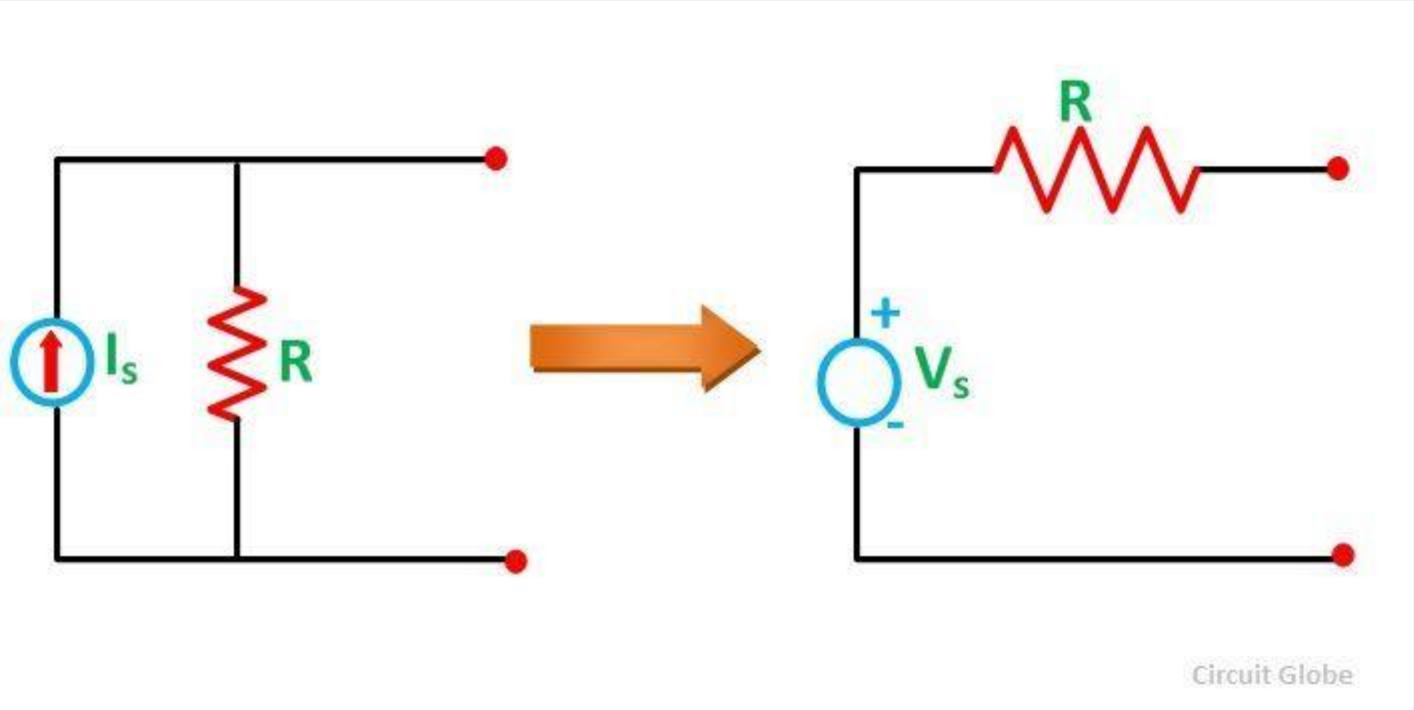


Practical voltage sources: Practical voltage sources always have some resistance value in series with an ideal voltage source and because of that series resistance, voltage drops when current passes through it. So, Practical Voltage Source has internal resistance and slightly variable voltage.

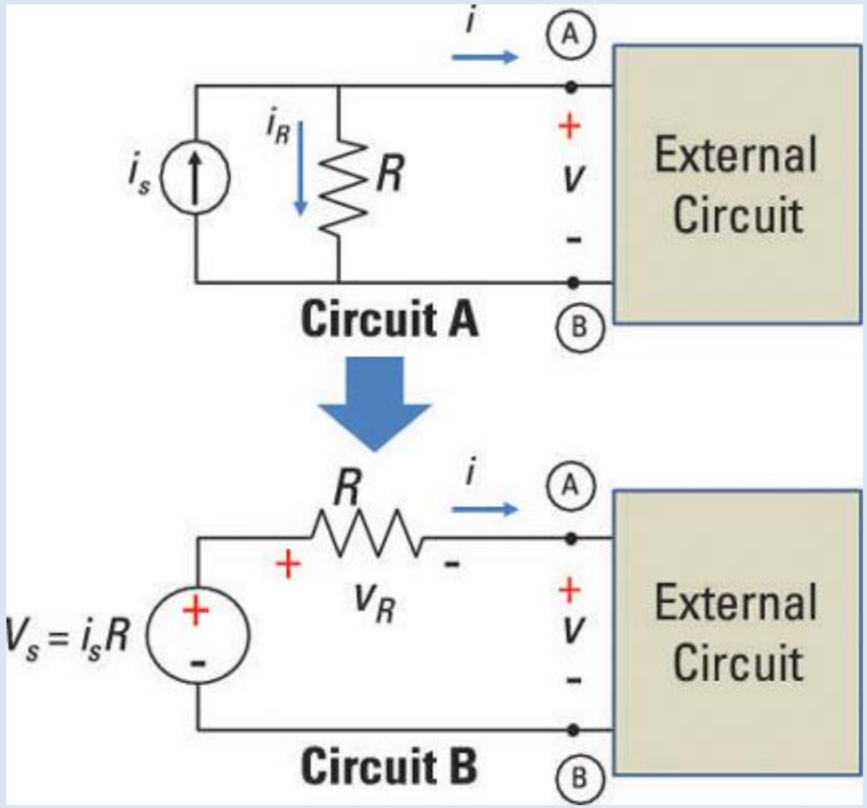
Source Transformation:



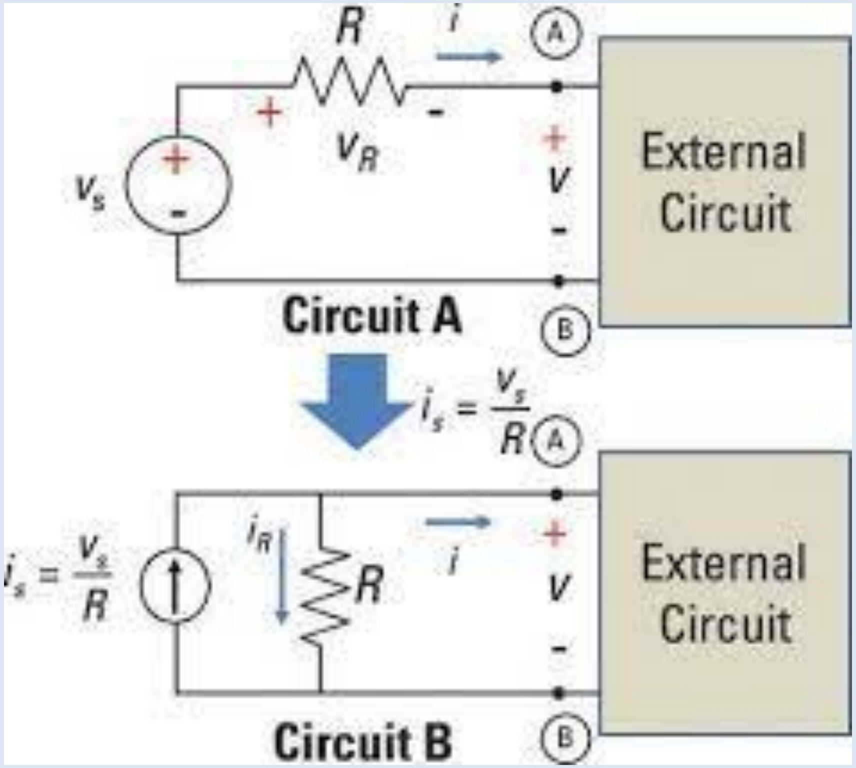


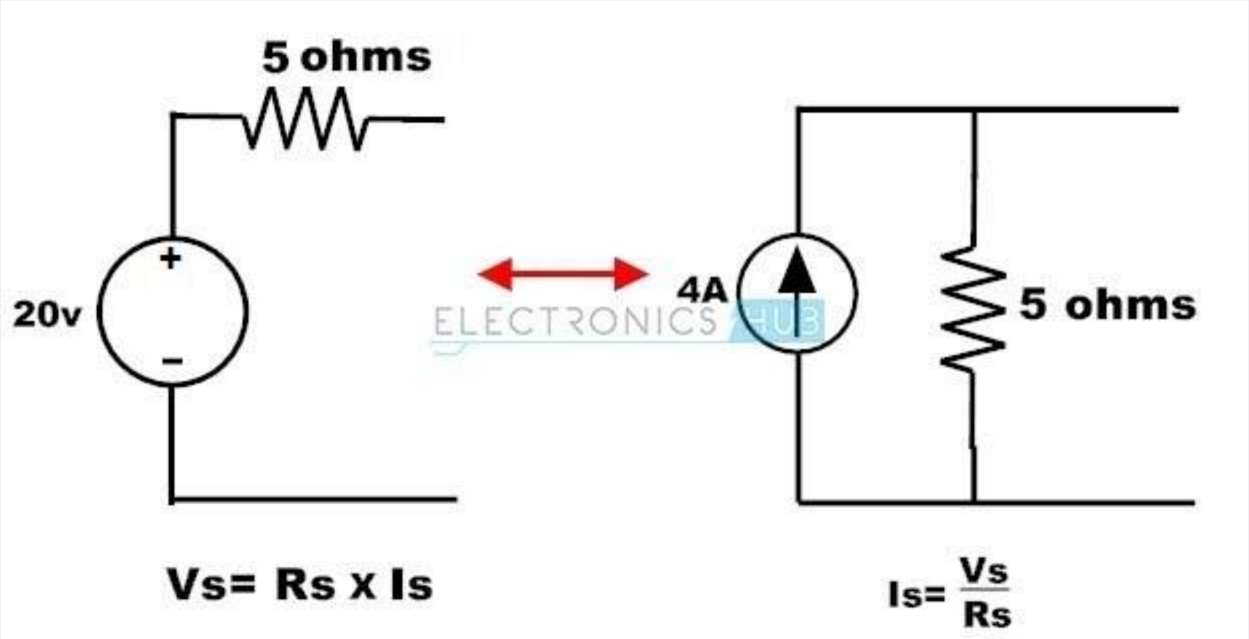


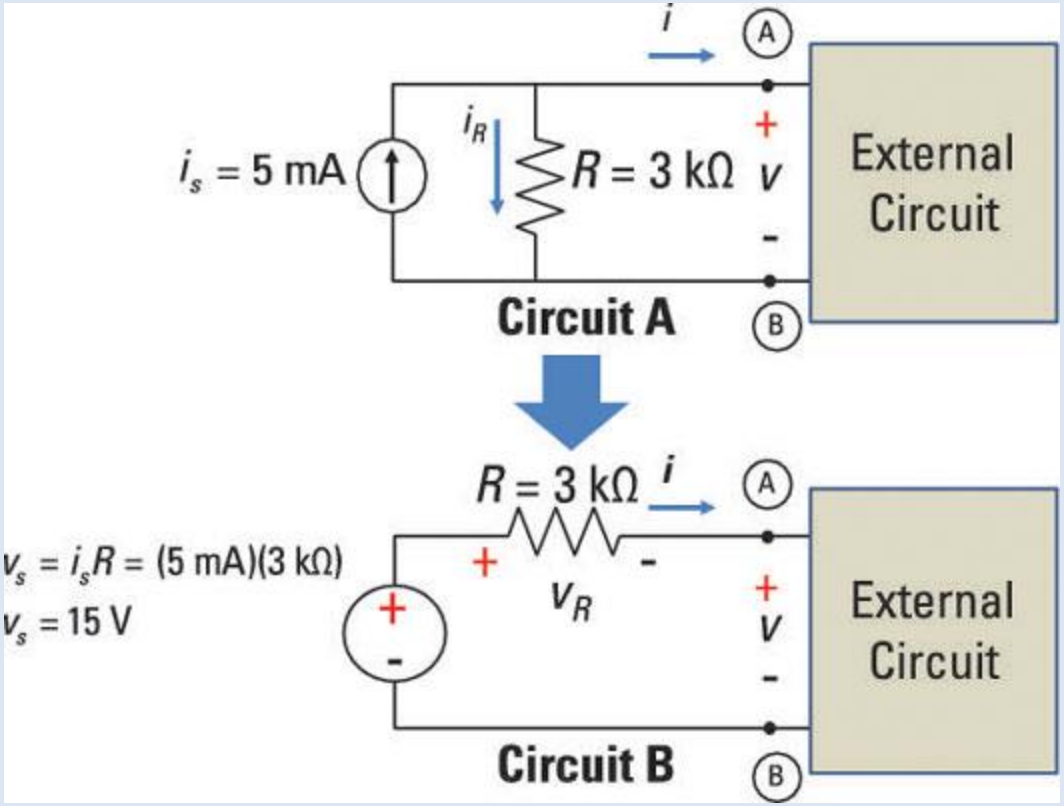
Is
R
R
+
Vs
-
Circuit Globe



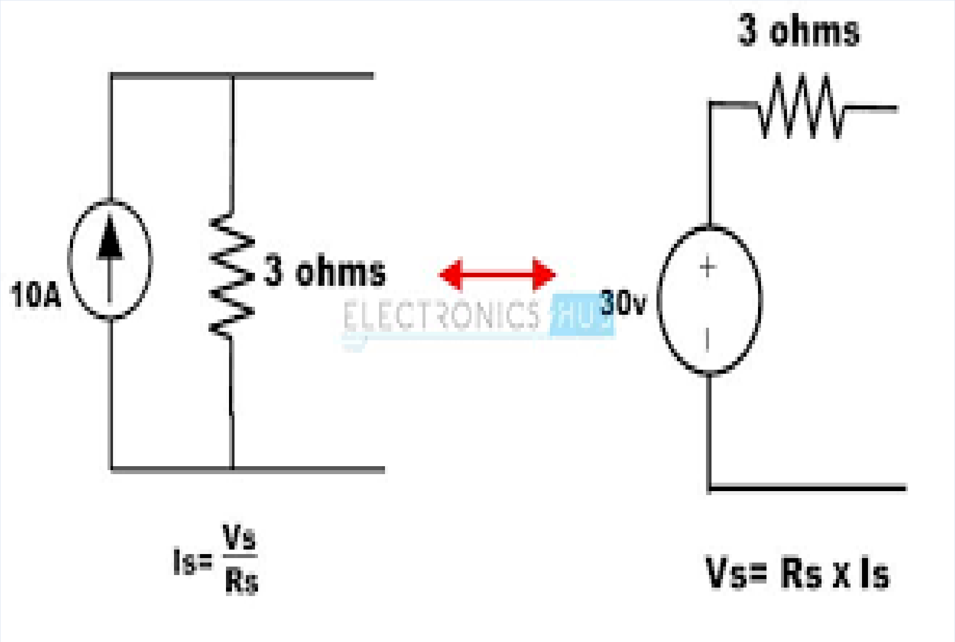
i
A
i_R
R
i_s
+
v
-
External
Circuit
Circuit A
B
R
i
A
+
-
v_R
+
V_s = i_sR
+
-
v
-
External
Circuit
Circuit B
B


R
i
A
+
-
v_R
v_s
+
v
-
External Circuit
Circuit A
B
$i_s = \frac{v_s}{R}$
A
i_R
R
i
+
v
-
$i_s = \frac{v_s}{R}$
External Circuit
Circuit B
B


5 ohms
20v
+
−
4A
5 ohms
ELECTRONICS HUB
Vs= Rs x Is
Is= Vs/Rs



i
A
i_R
i_s = 5 mA
R = 3 kΩ
+
v
-
External
Circuit
Circuit A
B
R = 3 kΩ
i
A
$v_s = i_sR$ = (5 mA)(3 kΩ)
v_s = 15 V
+
-
+
-
v_R
+
v
-
External
Circuit
Circuit B
B



3 ohms
10A
3 ohms
ELECTRONICS HUB
30v
+
Is= Vs/Rs
Vs= Rs x Is

ELECTRICAL NETWORKS

PARTS

Node

- A junction in a circuit where two or more circuit elements and/or branches are connected together.

Branch

- Part of a network which lies between two junctions.

Loop

- A closed path in a circuit in which no element or node is encountered more than once.

Mesh

- A loop that contains no other loop within it.

Circuit Definitions

Node – any point where 2 or more circuit elements are connected together

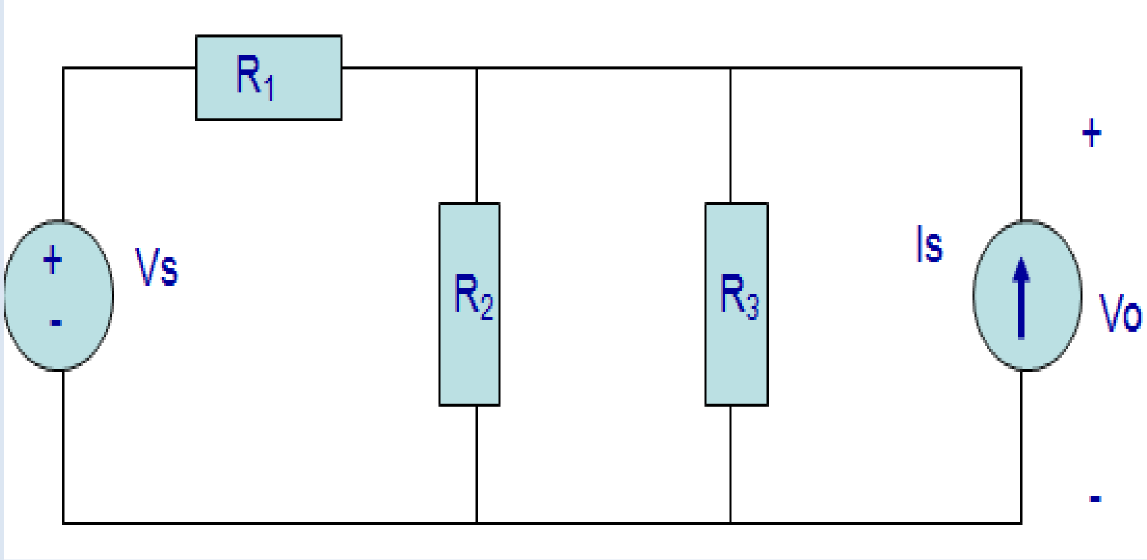
- Wires usually have negligible resistance
- Each node has one voltage (w.r.t. ground)

Branch – a circuit element between two nodes

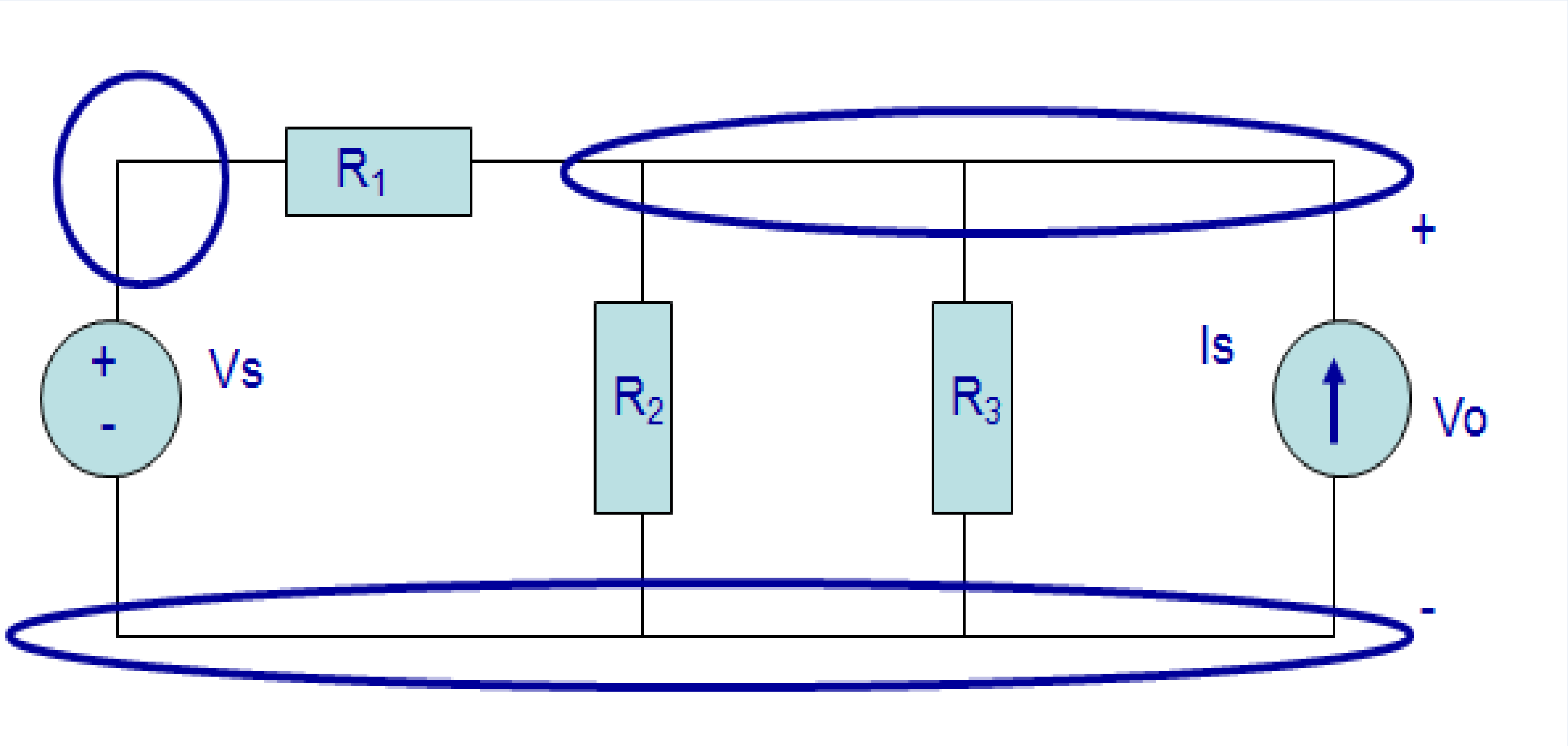
Loop – a collection of branches that form a closed path returning to the same node without going through any other nodes or branches twice

Example

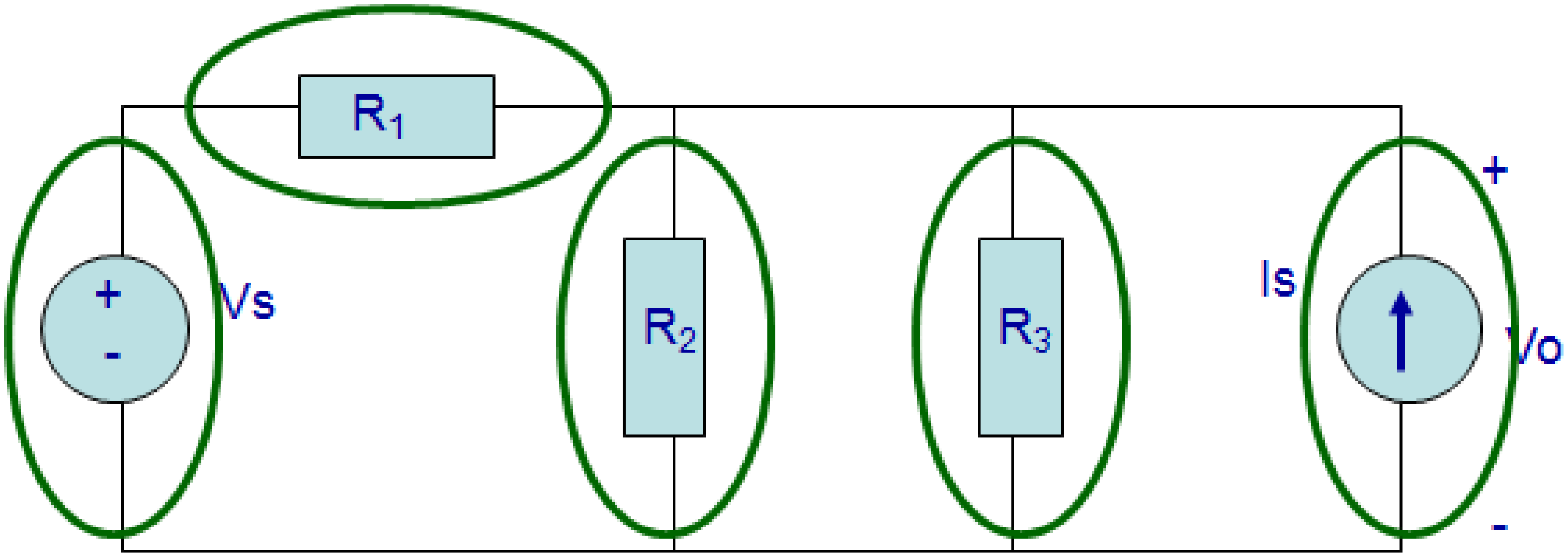
How many nodes, branches & loops?



Three nodes



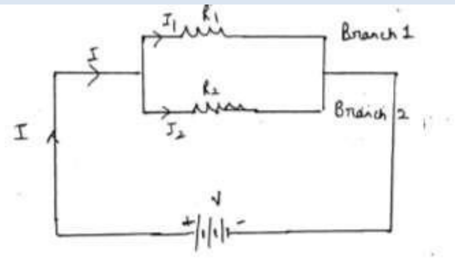
5 Branches



D. C. circuit: The closed path for flow of direct current is called D.C. circuit.

D.C Circuit is of two types:
1. Series Circuit
2. Parallel Circuit

Current in Parallel Circuit:



According to ohm's law

$$V = I_1 R_1 \quad \text{in branch 1}$$

$$V = I_2 R_2 \quad \text{" " 2}$$

$$V = I_1 R_1 = I_2 R_2$$

Total voltage in the circuit is $V = IR$ (ohm's law)

$$IR = I_1 R_1 = I_2 R_2$$

Total resistance in ||el circuit

$$R = \frac{R_1 R_2}{R_1 + R_2}$$

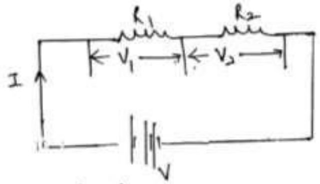
$$I_1 R_1 = I_2 R_2 = I\left(\frac{R_1 R_2}{R_1 + R_2}\right)$$

Then

$$I_1 = \frac{R_2}{R_1 + R_2}(I)$$

$$I_2 = \frac{R_1}{R_1 + R_2}(I)$$

Voltage in Series Circuit:



According to ohm's law

Current in resistance R_1 is

$$I = \frac{V_1}{R_1} \quad \text{——} \ (1)$$

Current in resistance R_2 is

$$I = \frac{V_2}{R_2} \quad \text{——} \ (2)$$

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = I \quad \text{——} \ (3)$$

Total current in circuit is

$$I = \frac{V}{R} \quad \text{——} \ (4)$$

Total resistance of the circuit is $R=R_1+R_2$

Put the values of R in eqn 3 and 4

$$\frac{V_1}{R_1} = \frac{V_2}{R_2} = \frac{V}{R_1+R_2}$$

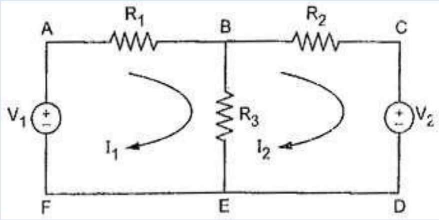
$$V_1 = \frac{R_1}{R_1+R_2}(V)$$

$$V_2 = \frac{R_2}{R_1+R_2}(V)$$

Network Terminology:

1. **Electric Network:**

Electric network is interconnection of electric components. E.g. Batteries, resistors, inductors and capacitors.



2. **Electric Circuit:**

The path for flow of electric current is called electric circuit.

3. **Active Elements:**

The elements which supplies energy to the circuit. In fig V_1 and V_2 are active elements.

4. **Passive Elements:**

The elements which receives energy. In fig R_1, R_2 and R_3 are passive elements.

5. **Node:**

Node is a point where two or more circuit elements are connected together. In Fig. A, B, C and E are nodes.

6. **Junction:**

Junction is a point in the network where three or more circuit elements are connected together. It is a point where current is divided. In Fig. B and E are junctions.

7. **Loop:**

The closed path of a network. E.g. ABEFA, BCDEB and ABCDEFA are loops.

8. **Mesh:**

The elementary form of loop which cannot be further divided is called mesh.
E.g. ABEFA, BCDEB are mesh.

9. **Branch:**

Part of a network which lies between two junction points. In fig. ABEFA, BCDEB AND BE are the three branches.

SERIES CIRCUITS

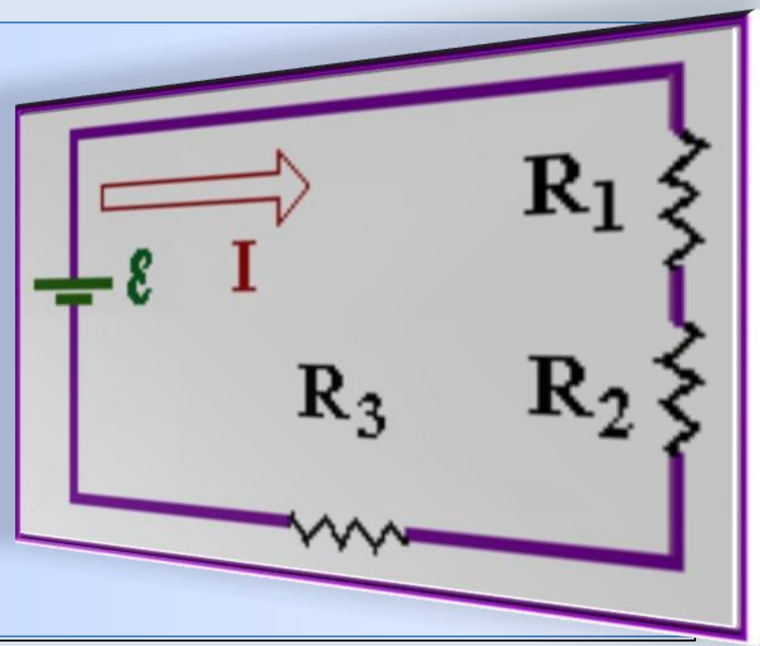
SERIES circuits

- A circuit connection in which the components are connected to form one conducting path

$$R_{equivalent} = R_1 + R_2 + R_3$$

$$V_{total} = V_1 + V_2 + V_3$$

$$I = I_1 = I_2 = I_3$$



SERIES CIRCUITS

Voltage Division for Series Circuit:

$$E_X = E_T \cdot \frac{R_X}{R_T}$$

Where: E_X – voltage across the resistor concerned

E_T – total voltage across the circuit

R_X – the resistor concerned

R_T – the sum of all resistances in the circuit

PARALLEL CIRCUITS

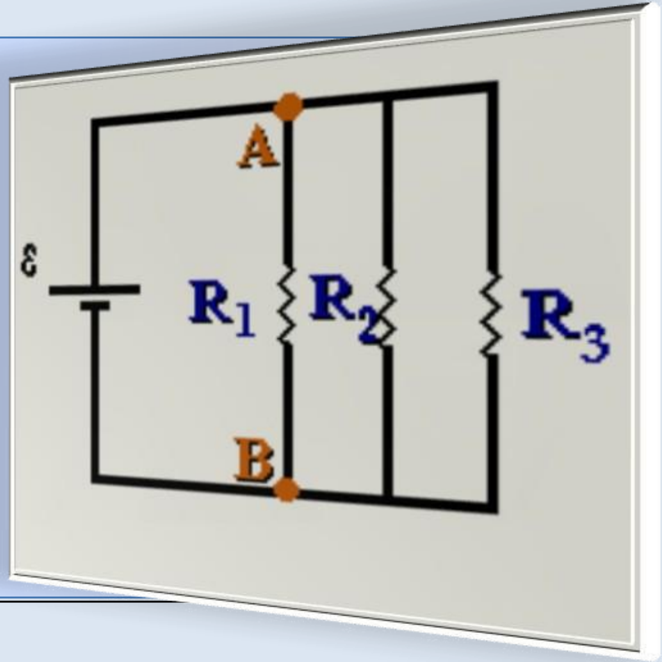
PARALLEL circuits

- A circuit connection in which the components are connected to form more than 1 conducting path

$$\frac{1}{R_{equivalent}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

$$V_1 = V_2 = V_3 = V$$

$$I = I_1 + I_2 + I_3$$



PARALLEL CIRCUITS

Voltage Division for Parallel Circuit:

$$I_X = I_T \bullet \frac{R_{eq}}{R_T}$$

Where: I_X – current concerned flowing through resistor R_x I_T

– total current of the circuit

R_{eq} – equivalent resistance of the parallel circuit except R_x

R_T – the sum of all resistances in the circuit

KIRCHHOFF’S LAW

KIRCHHOFF’S LAW

- More comprehensive than Ohm’s Law and is used in solving electrical
- Termed as “Laws of Electric Networks”
- Formulated by German physicist **Gustav Robert Kirchhoff**

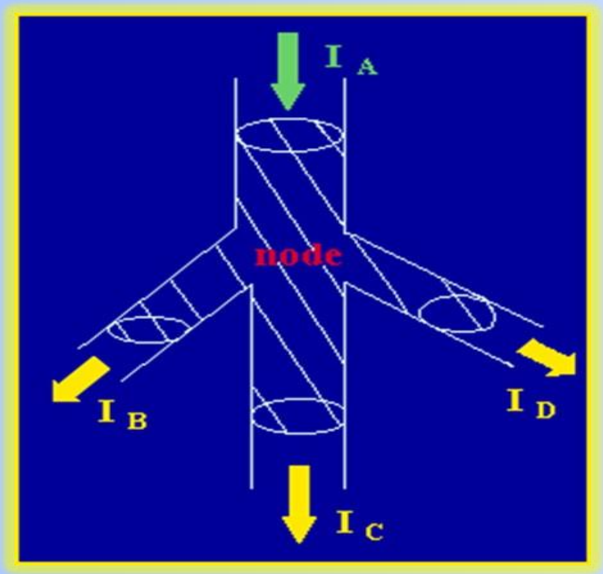
Kirchhoff’s Current Law (KCL)

“In any electrical network, the algebraic sum of the current meeting at a point (or junction) is zero.”

$$\sum I = 0$$

KIRCHHOFF'S CURRENT LAW

- In short the sum of currents entering a node equals the sum of currents leaving the node
  - Current towards the node, positive current
  - Current away from the node, negative current



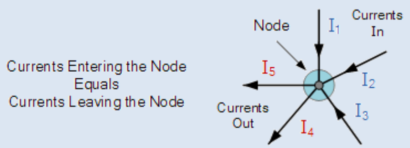
$$I_B + I_C + I_D = I_A$$

$$(I_B + I_C + I_D) - I_A = 0$$

Kirchhoff’s Current Law or KCL

Kirchhoff‟s Current Law or KCL, states that the “total current or charge entering a junction or node is exactly equal to the charge leaving the node”. In other words the algebraic sum of all the currents entering and leaving a node must be equal to zero, I(exiting) + I(entering) = 0.

Kirchhoff‟s Current Law or KCL



$$I_1 + I_2 + I_3 + (-I_4 + -I_5) = 0$$

Here, the three currents entering the node, I_1, I_2, I_3 are all positive in value and the two currents leaving the node, I_4 and I_5 are negative in value. Then this means we can also rewrite the equation as;

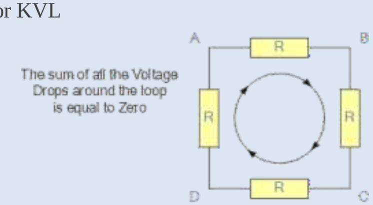
$$I_1 + I_2 + I_3 - I_4 - I_5 = 0$$

The term Node in an electrical circuit generally refers to a connection or junction of two or more current carrying paths or elements such as cables and components. Also for current to flow either in or out of a node a closed circuit path must exist. We can use Kirchhoff‟s current law when analyzing parallel circuits.

Kirchhoff’s Voltage Law

Kirchhoff‟s Voltage Law or KVL, states that “in any closed loop network, the total voltage around the loop is equal to the sum of all the voltage drops within the same loop” which is also equal to zero. In other words the algebraic sum of all voltages within the loop must be equal to zero.

Kirchhoff‟s Voltage Law or KVL

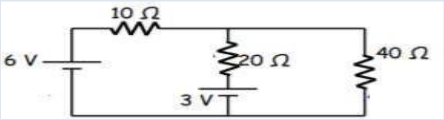


$$V_{AB} + V_{BC} + V_{CD} + V_{DA} = 0$$

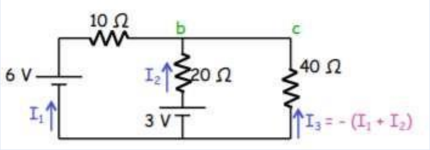
Starting at any point in the loop continue in the same direction noting the direction of all the voltage drops, either positive or negative, and returning back to the same starting point. It is important to maintain the same direction either clockwise or anti-clockwise or the final voltage sum will not be equal to zero. We can use Kirchhoff‟s voltage law when analyzing series circuits.

Analysis of simple circuits with Kirchhoff's law

Q: Calculate current in given circuit using Kirchhoff's law.



Ans: Firstly we have to mark the direction of current in given circuit.



Junction b: $I_1 + I_2 + I_3 = 0$

So, $I_3 = -(I_1 + I_2)$

Loop abcda: $6\,V - I_1 \cdot 10\Omega + 40\Omega(-I_1 - I_2) = 0$

$\Rightarrow 50I_1 + 40I_2 = 6$ ..... (1)

Loop dbcd: $3\,V - 20\Omega \cdot I_2 + 40\Omega(-I_1 - I_2) = 0$

$\Rightarrow 40I_1 + 60I_2 = 3$ ..... (2)

4×(1) - 5×(2) : $-140\, I_2 = 9$ → $I_2 = -64$ mA

3×(1) - 2×(2) : $70\, I_1 = 12$ → $I_1 = 171$ mA

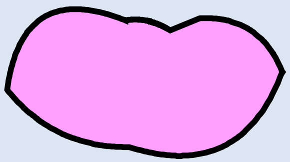
$I_3 = -(171-64) = -107$mA

$I_3 = -107$mA

Basic Laws of Circuits

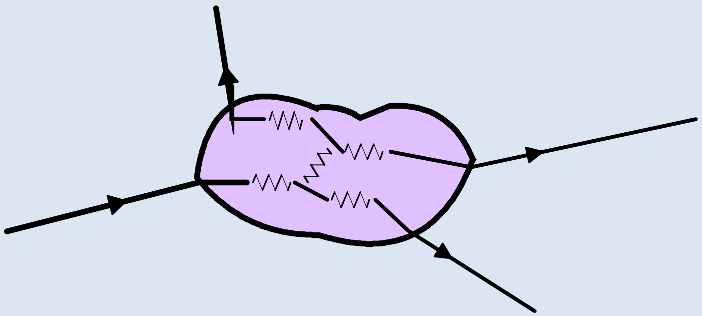
Kirchhoff's Current Law

Kirchhoff's current law can be generalized to include a surface.
We assume the elements within the surface are interconnected.



A closed 3D surface

We can now apply Kirchhoff's current law in the 3 forms we discussed with a node. The appearance might be as follows:



Currents entering and leaving a closed surface that contains interconnected circuit elements

KIRCHHOFF'S VOLTAGE LAW

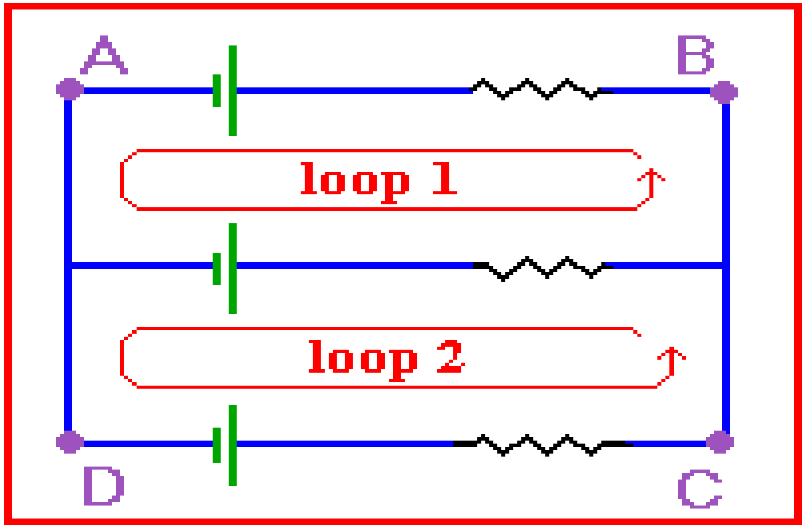
Kirchhoff's Voltage Law (KVL)

"The algebraic sum of the products of currents and resistances in each of thr conductors in any closed path (or mesh) in a network PLUS the algebraic sum of the emfs in the path is zero."

$$\sum V = 0$$

KIRCHHOFF'S VOLTAGE LAW

- In short, the sum of the voltages around the loop is equal to zero
  - For voltage sources, if loops enters on minus and goes out on plus, positive voltage and if loops enters on plus and goes out on minus, negative voltage.
  - For voltage drops, if the loop direction is the same as current direction, negative voltage drop and if the loop direction is opposite to the current direction, positive voltage drop.



Kirchoff’s Voltage Law (KVL)

The algebraic sum of voltages around each loop is zero

Beginning with one node, add voltages across each branch in the loop

(if you encounter a + sign first) and subtract voltages (if you encounter a – sign first)

Σ voltage drops - Σ voltage rises = 0

Or Σ voltage drops = Σ voltage rises

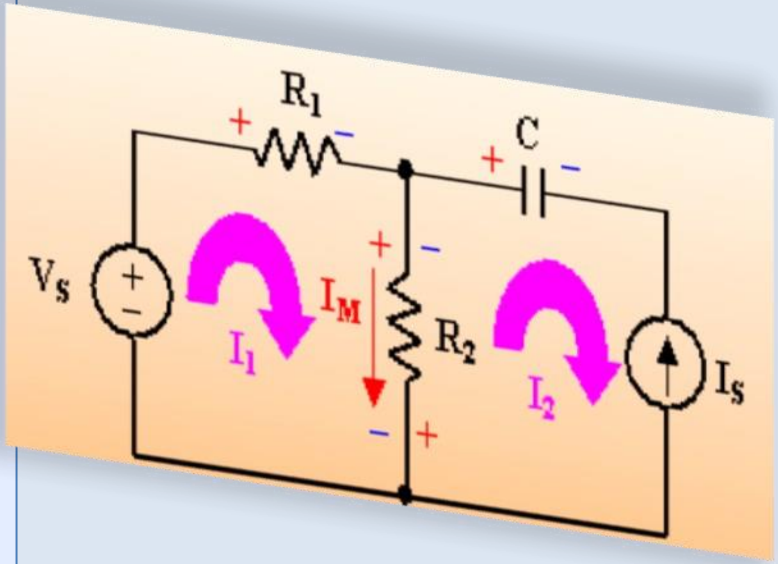
MESH ANALYSIS

MESH analysis

- A sophisticated application of KVL with mesh currents.

Loop Analysis Procedure:

1. **Label each of the loop/mesh currents.**
2. **Apply KVL to loops/meshes to form equations with current variables.**
   a. **For *N* independent loops, we may write *N* total equations using KVL around each loop. *Loop currents* are those currents flowing in a loop; they are used to define *branch currents*.**
   b. **Current sources provide constraint equations.**
3. **Solve the equations to determine the user defined loop currents.**



NODAL ANALYSIS

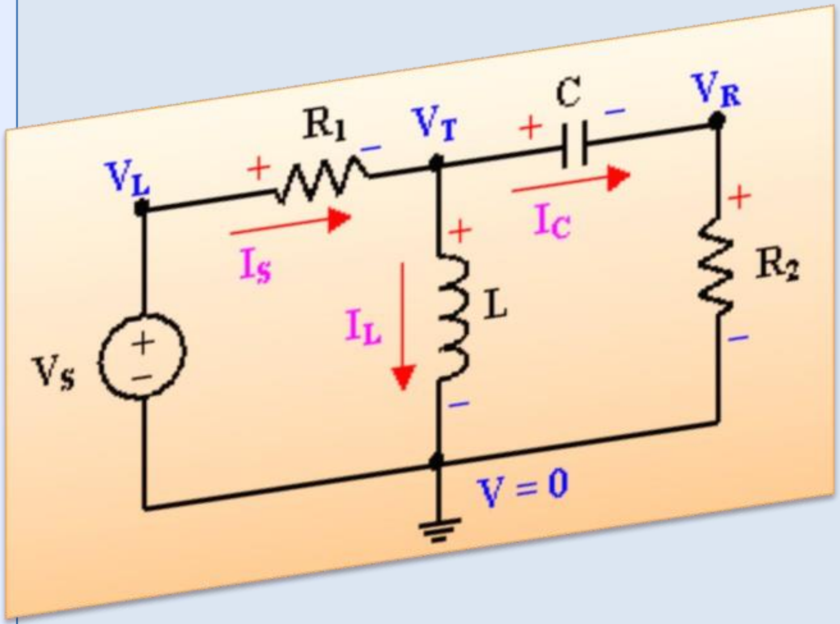
NODAL analysis

- A systematic application of KCL at a node and after simplifying the resulting KCL equation, the node voltage can be calculated.

- **Consist of finding the node voltages at all principal nodes with respect to the reference node.**

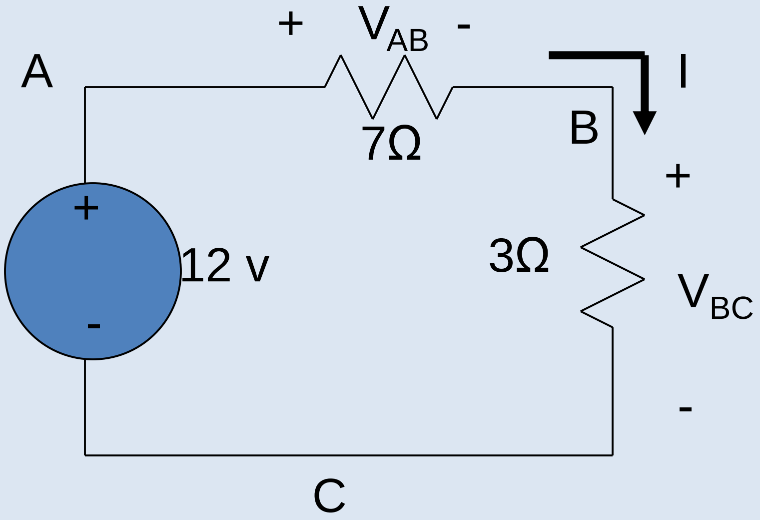
PRINCIPAL node – a node with three or more circuit elements joined together.

Reference node – the node from which the unknown voltages are measured.



Circuit Analysis

When given a circuit with sources and resistors having fixed values, you can use Kirchoff's two laws and Ohm's law to determine all branch voltages and currents



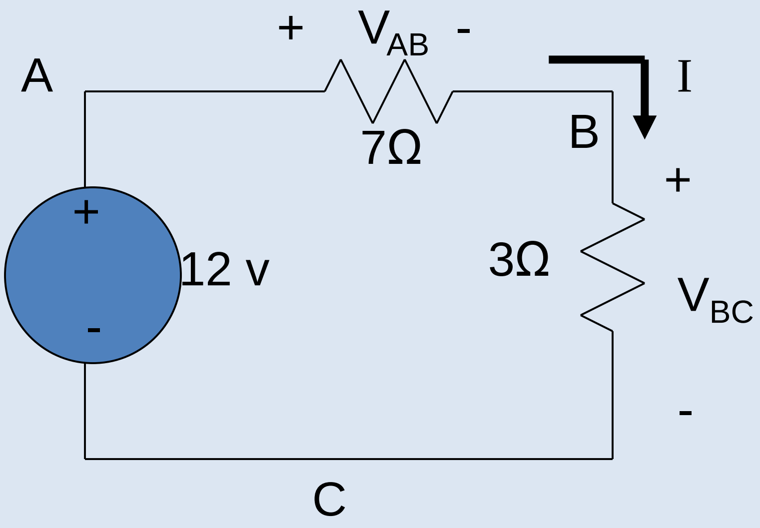
Circuit Analysis

By Ohm's law: $V_{AB} = I \cdot 7\Omega$ and $V_{BC} = I \cdot 3\Omega$

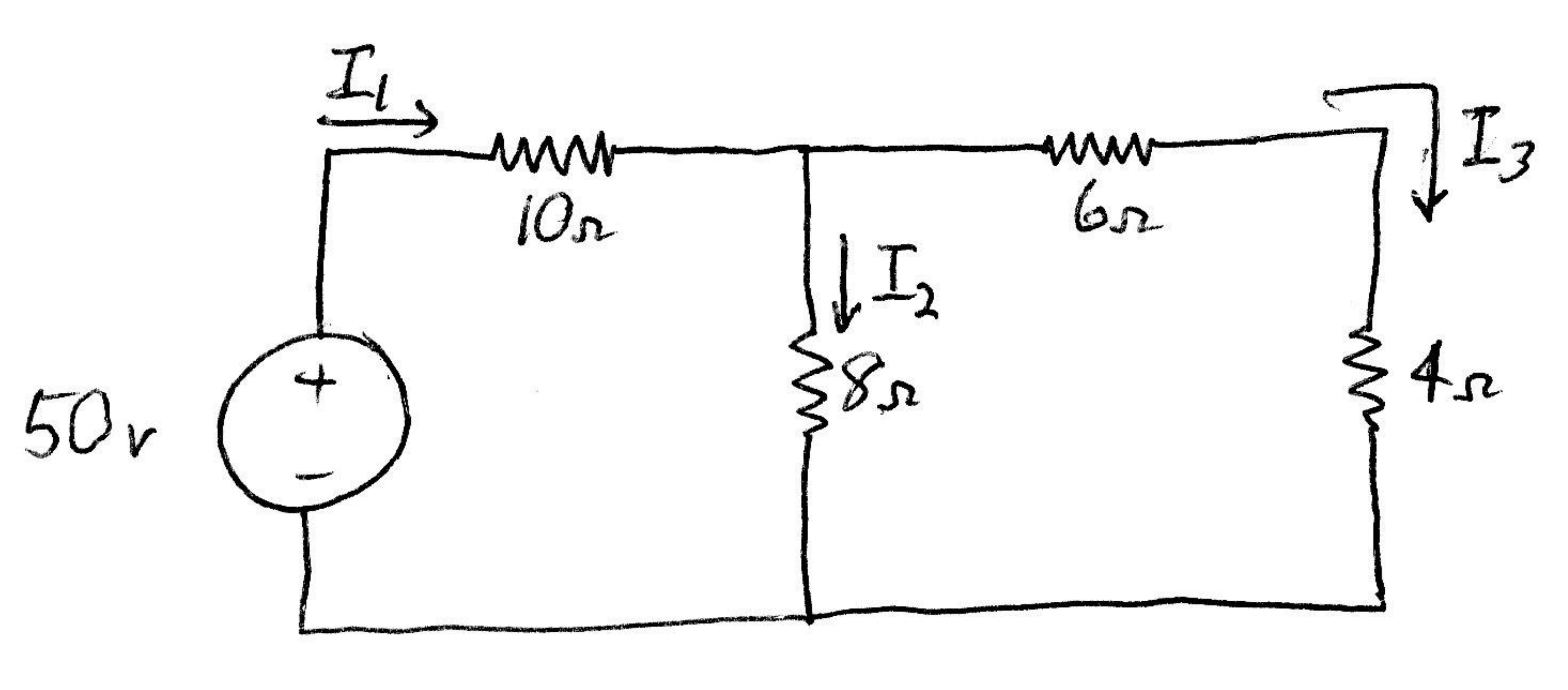
By KVL: $V_{AB} + V_{BC} - 12\text{ v} = 0$

Substituting: $I \cdot 7\Omega + I \cdot 3\Omega - 12\text{ v} = 0$

Solving: $I = 1.2\text{ A}$

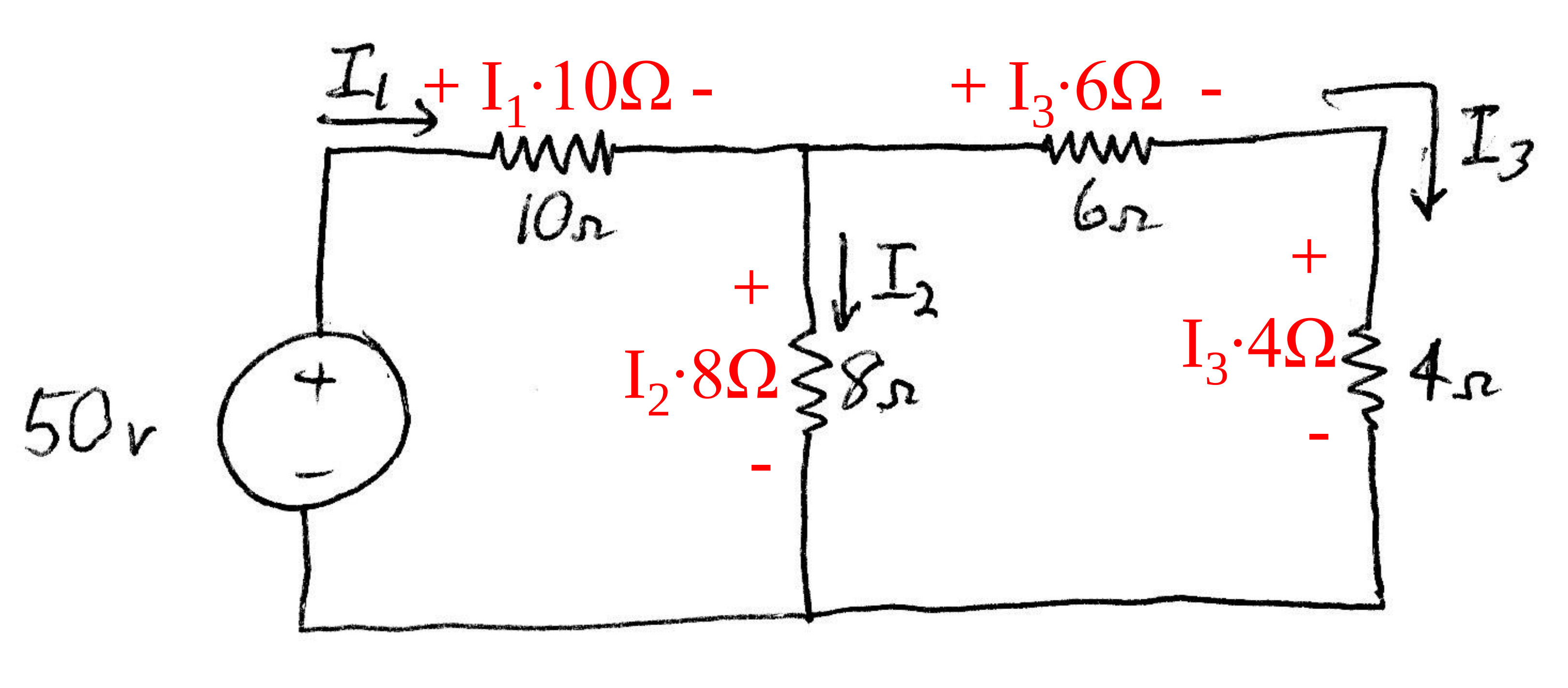


Example Circuit



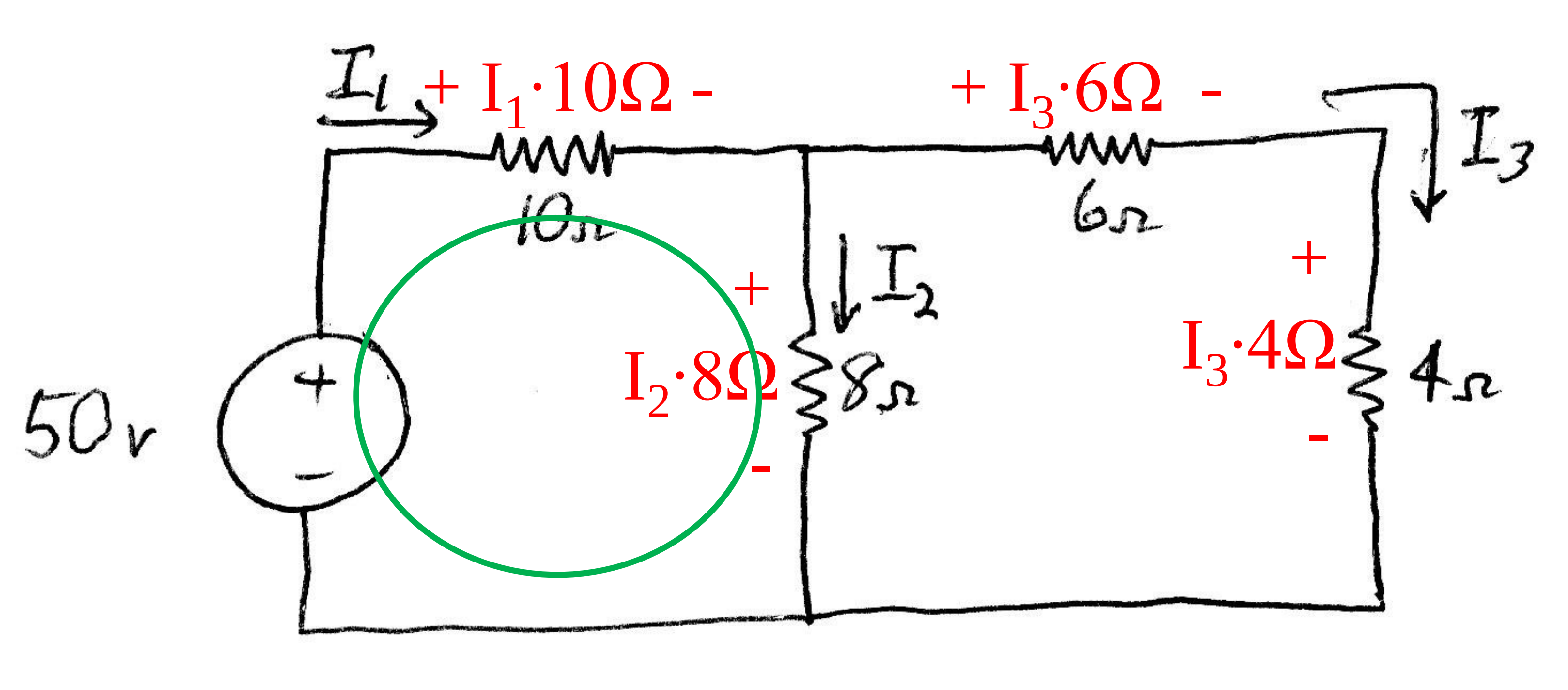
Solve for the currents through each resistor
And the voltages across each resistor

Example Circuit



Using Ohm's law, add polarities and expressions for each resistor voltage

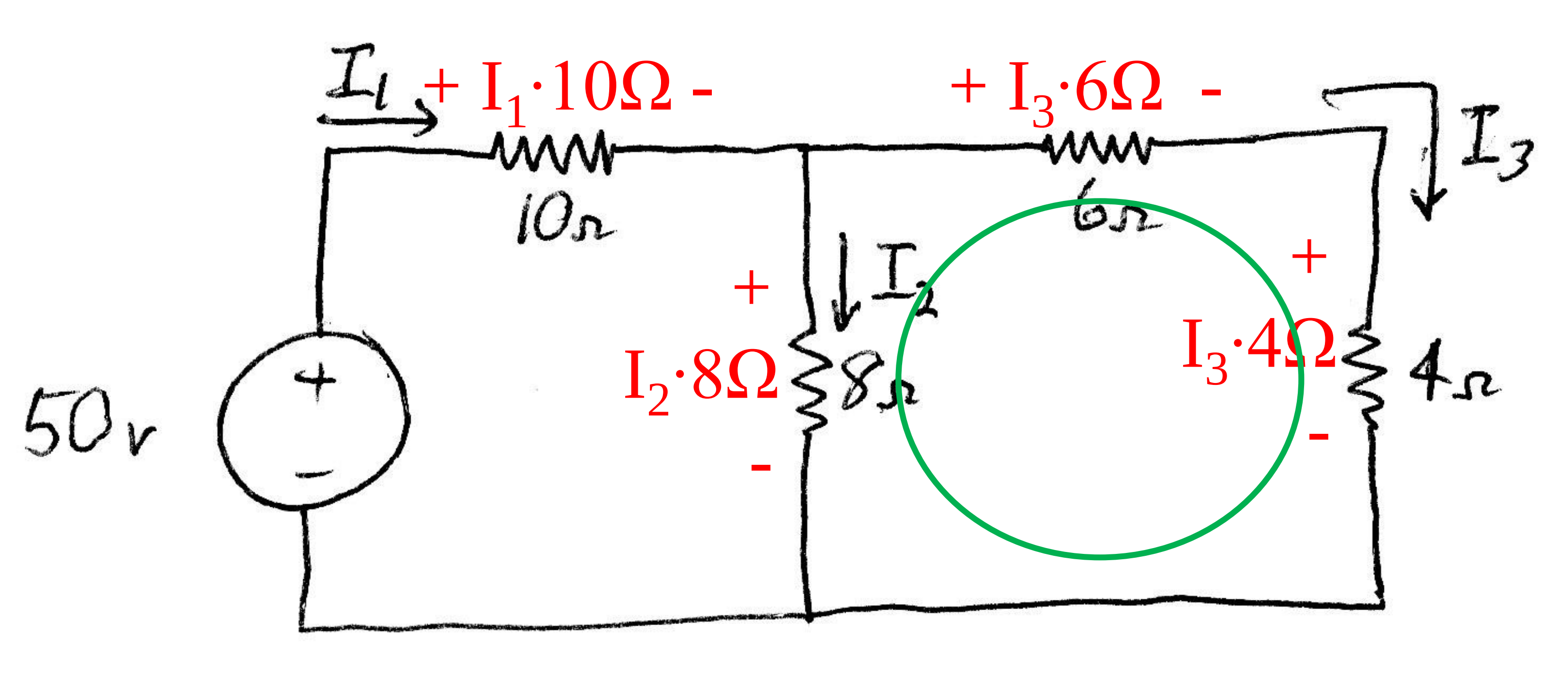
Example Circuit



Write 1st Kirchoff's voltage law equation

$$-50\ \text{v} + I_1 \cdot 10\Omega + I_2 \cdot 8\Omega = 0$$

Example Circuit



Write 2nd Kirchoff's voltage law equation

$$-I_2 \cdot 8\Omega + I_3 \cdot 6\Omega + I_3 \cdot 4\Omega = 0$$

$$\text{or } I_2 = I_3 \cdot (6+4)/8 = 1.25 \cdot I_3$$

Example Circuit

We now have 3 equations in 3 unknowns, so we can solve for the currents through each resistor, that are used to find the voltage across each resistor

Since $I_1 - I_2 - I_3 = 0$, $I_1 = I_2 + I_3$

Substituting into the 1st KVL equation

$$-50\text{ v} + (I_2 + I_3)\cdot 10\Omega + I_2\cdot 8\Omega = 0$$

$$\text{or } I_2\cdot 18\ \Omega + I_3\cdot 10\ \Omega = 50\text{ volts}$$

Example Circuit

But from the 2nd KVL equation, $I_2 = 1.25 \cdot I_3$

Substituting into 1st KVL equation:

$(1.25 \cdot I_3) \cdot 18\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 22.5\ \Omega + I_3 \cdot 10\ \Omega = 50$ volts

Or: $I_3 \cdot 32.5\ \Omega = 50$ volts

Or: $I_3 = 50$ volts$/32.5\ \Omega$

Or: $I_3 = 1.538$ amps

Example Circuit

Since $I_3 = 1.538$ amps

$I_2 = 1.25 \cdot I_3 = 1.923$ amps

Since $I_1 = I_2 + I_3$, $I_1 = 3.461$ amps

The voltages across the resistors:

$I_1 \cdot 10\Omega = 34.61$ volts

$I_2 \cdot 8\Omega = 15.38$ volts

$I_3 \cdot 6\Omega = 9.23$ volts

$I_3 \cdot 4\Omega = 6.15$ volts

SUPERPOSITION THEOREM

SUPERPOSITION theorem

" The current through or voltage across, an element in a linear bilateral network is equal to the algebraic sum of the current or voltages produced independently in each source. "

In general:

- Number of network to analyze is equal to number of independent sources.
- To consider effects of each source independently, sources must be removed and replaced without affecting the final result:

✓ All voltage sources >> short circuited

✓ All current sources >> open circuited

<u>Superposition theorem</u>

Superposition theorem states that in any linear, active, bilateral network having more than one source, the response across any element is the sum of the responses obtained from each source considered separately and all other sources are replaced by their internal resistance. The superposition theorem is used to solve the network where two or more sources are present and connected. Resulting current in any branch is the algebraic sum of all the currents that would be produced in it.

Procedure for using superposition theorem

Step-1: Retain one source at a time in the circuit and replace all other sources with their internal resistances.

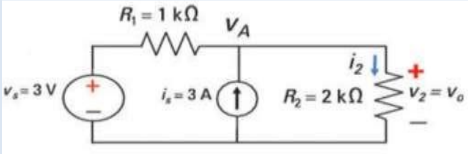
Step-2: Determine the output (current or voltage) due to the single source acting alone using the techniques discussed in lessons 3 and 4.

Step-3: Repeat steps 1 and 2 for each of the other independent sources.

Step-4: Find the total contribution by adding algebraically all the contributions due to the independent sources

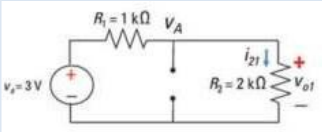
Numerical

Calculate the total voltage in given fig using Superposition theorem



We need to turn off the independent sources one at a time. To do so, replace the current source with an open circuit and the voltage source with a short circuit.

Considering the voltage source

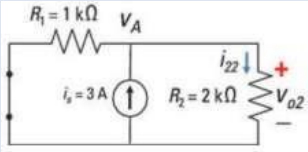


We can use the voltage divider technique because its resistors, R_1 and R_2, are connected in series with a voltage source. So here‟s the voltage v_{o1} across resistor R_2:

$$v_{o1} = \left(\frac{R_2}{R_1 + R_2}\right) v_s$$

$$v_{o1} = \left(\frac{2\ \text{k}\Omega}{1\ \text{k}\Omega + 2\ \text{k}\Omega}\right)(3\ \text{V}) = 2\ \text{V}$$

Considering Current source



We can use a current divider technique because the resistors are connected in parallel with a current source. The current source provides the following current i_{22} flowing through resistor R_2:

$$i_{22} = \left(\frac{R_1}{R_1 + R_2}\right) i_s$$

$$i_{22} = \left(\frac{1\ \text{k}\Omega}{1\ \text{k}\Omega + 2\ \text{k}\Omega}\right)(3\ \text{A}) = 1\ \text{mA}$$

We can use Ohm‟s law to find the voltage output v_{o2} across resistor R_2

$$v_{o2} = i_{22} R_2$$

$$v_{o2} = (1\ \text{mA})(2\ \text{k}\Omega) = 2\ \text{V}$$

Now find the total output voltage across R_2 for the two independent sources in Circuit C by adding v_{o1} (due to the source voltage v_s) and v_{o2} (due to the source current i_s). You wind up with the following output voltage.

$v_o = v_{o1} + v_{o2}$

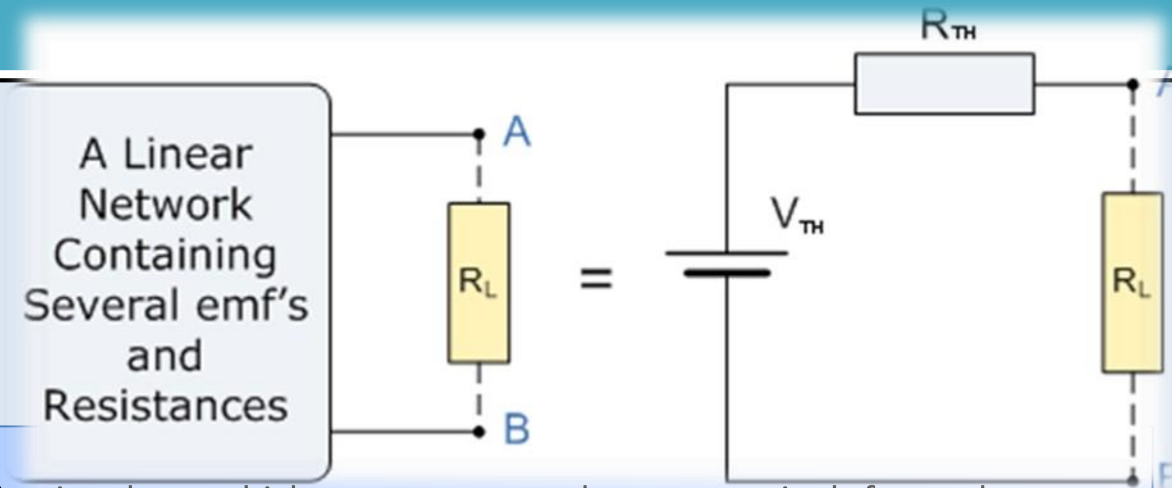
$v_o = 2+2 = 4$ V

.

THEVENIN'S THEOREM

THEVENIN'S theorem

" Any two-terminal of a linear, active bilateral network of a fixed resistances and voltage source/s may be replaced by a single voltage source (**V_{TH}**) and a series of internal resistance (**R_{TH}**). "



where:

V_{TH} – the open circuit voltage which appears across the two terminals from where the load resistance has been removed.

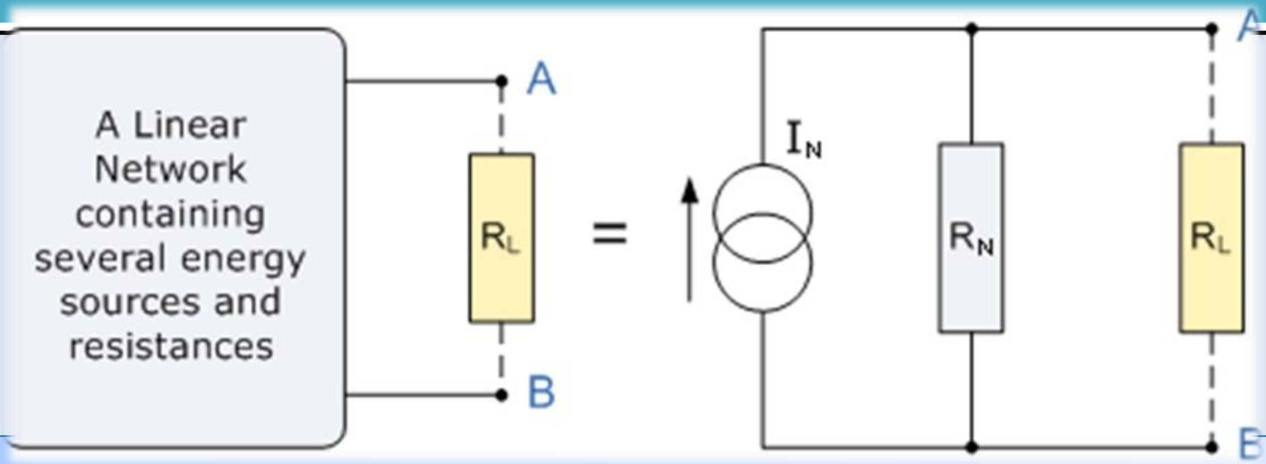
R_{TH} – the resistance looking back into the network across the two terminals with all the voltage sources shorted and replaced by their internal resistances (if any) and all current sources by infinite resistance.

c re u by

NORTON'S THEOREM

THEVENIN'S theorem

" Any two-terminal active network containing voltage sources and resistances when viewed from its output terminals, is equivalent to a constant-current source (**I_N**) and a parallel internal resistance (**R_N**). "

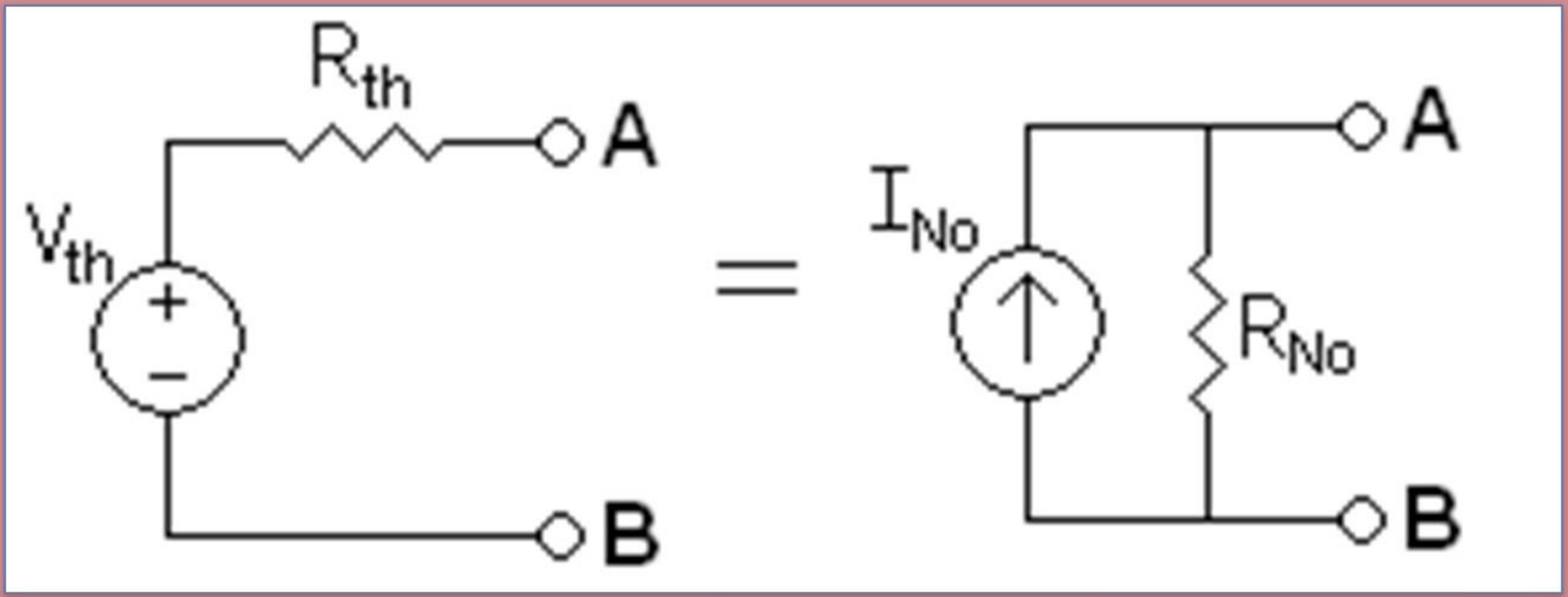


where:

I_N – the current which would flow in a short circuit placed across the output terminals.

R_N – the resistance of the network when viewed from the open circuited terminals after all voltage sources being replaced by open circuits.

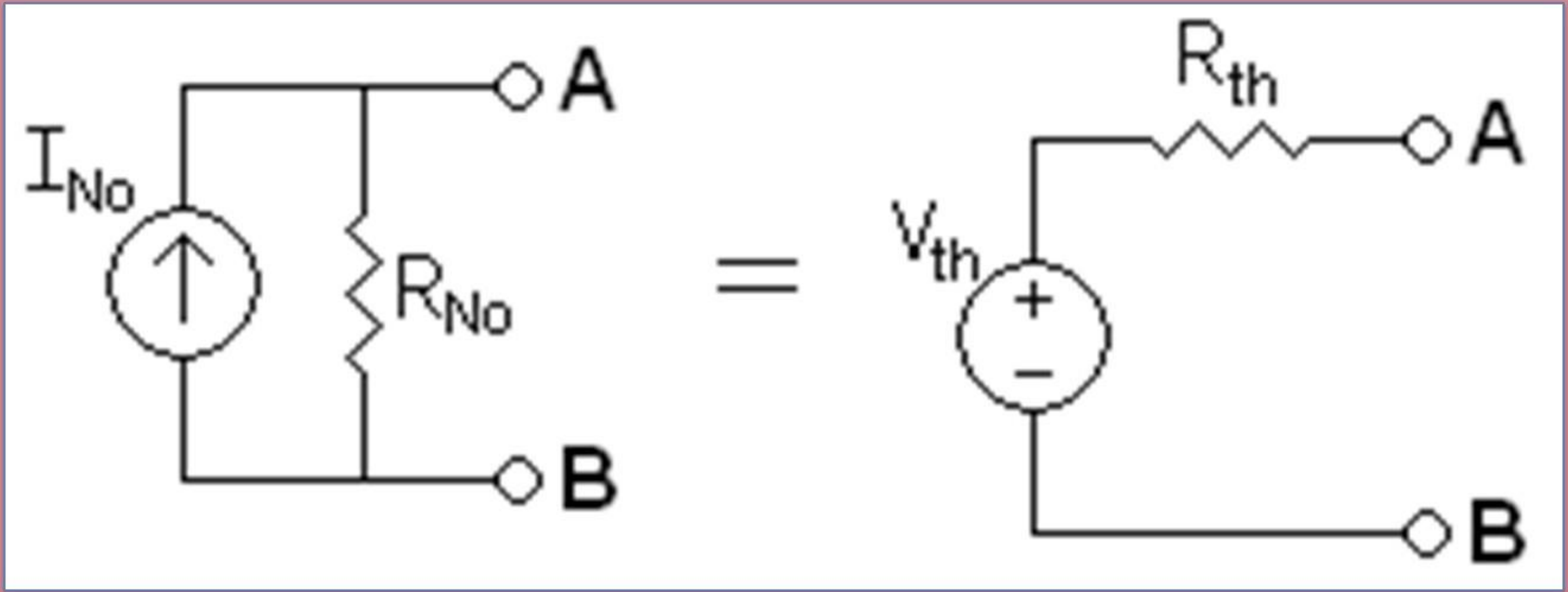
THEVENIN-NORTON TRANSFORMATION



$$I_{Norton} = \frac{E_{Thevenin}}{R_{Thevenin}}$$

$$R_{Thevenin} = R_{Norton}$$

NORTON-THEVENIN TRANSFORMATION



$$E_{Thevenin} = I_{Norton} R_{Norton}$$

$$R_{Thevenin} = R_{Norton}$$

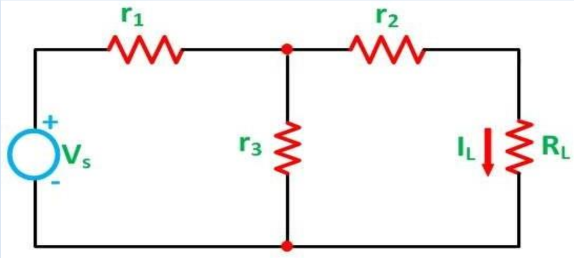
NETWORK ANALYSIS

Norton's Theorem

Statement:
Norton‟s Theorem states that – A linear active network consisting of the independent or dependent voltage source and current sources and the various circuit elements can be substituted by an equivalent circuit consisting of a current source in parallel with a resistance.
The current source being the short-circuited current across the load terminal and the resistance being the internal resistance of the source network.
The Norton‟s theorems reduce the networks equivalent to the circuit having one current source, parallel resistance and load. Norton‟s theorem is the converse of Thevenin‟s Theorem. It consists of the equivalent current source instead of an equivalent voltage source as in Thevenin‟s theorem.

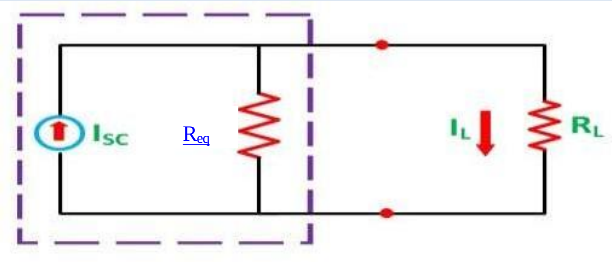
Explanation of Norton's Theorem

To understand Norton‟s Theorem in detail, let us consider a circuit diagram given below



In order to find the current through the load resistance I_L as shown in the circuit diagram above

Step 1:



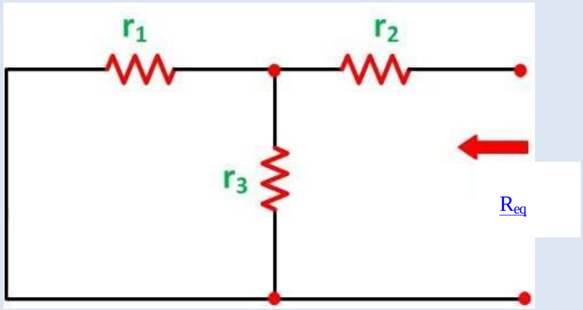
$I_L = I_{SC} \frac{Req}{Req + RL}$

Where, I_L is the load current, I_{sc} is the short circuit current

R_{eq} is the equivalentl resistance of the circuit, R_L is the load resistance of the circuit

Step 3: To find Req

Now the short circuit is removed, and the independent source is deactivated as shown in the circuit diagram below and the value of the equivalent resistance is calculated by:

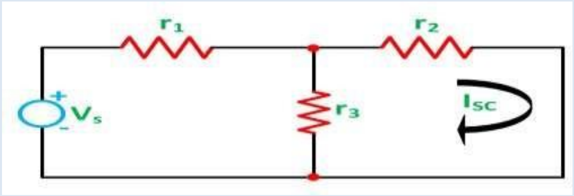


So,

$Req = r_1 \| r_3 + r_2$

$Req = \frac{r1r3}{r1+r3} + r_2$

Step 2: To find Isc



Now, the value of current I flowing in the circuit is found out by the equation

$$I = \frac{V_S}{r_1 + \frac{r_2 r_3}{r_2 + r_3}}$$

And the short-circuit current I_{SC} is given by the equation shown below:

$$I_{sc} = I \frac{r_3}{r_3 + r_2}$$

Steps for Solving a Network Utilizing Norton's Theorem

Step 1 – Norton‟s equivalent circuit is drawn by keeping the equivalent resistance R_{eq} in parallel with the short circuit current I_{SC}.

Step 2 – Find the internal resistance R_{eq} of the source network by deactivating the constant sources.

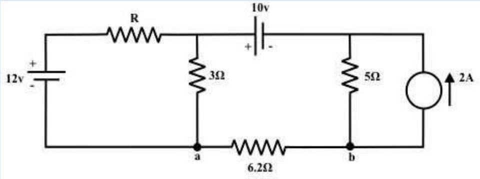
Step 3 – Now short the load terminals and find the short circuit current I_{SC} flowing through the shorted load terminals using conventional network analysis methods.

Step 4 – Reconnect the load resistance R_L of the circuit across the load terminals and find the current through it known as load current $I_{L.}$

This is all about Norton‟s Theorem.

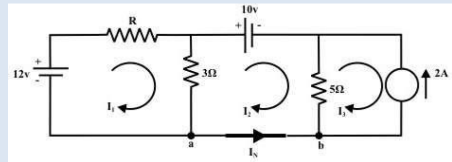
Numerical

Applying Norton‟s theorem, find the maximum power dissipated by the resistor 6.2Ω under that situation



Solution of numerical

Step-1: Short the terminals „a‟ and „b‟ after disconnecting the 6.2 resistor. The Norton‟s current I_N for the circuit shown in fig. is computed by using „mesh-current‟ method.



Loop-1:

$$12 - I_1 R - 3(I_1 - I_2) = 0$$

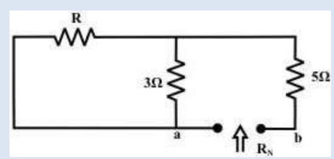
Loop-2:

$$-10 - 5(I_2 - I_3) - 3(I_2 - I_1) = 0, \quad note \;\; I_3 = -2\,A$$

Solving equations

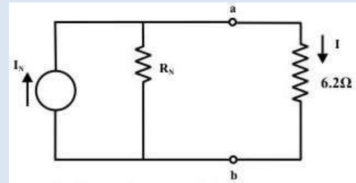
$I_1 = \dfrac{36}{15+8R}$; $I_2 = -\dfrac{24+20R}{15+8R}$ (-ve sign implies that the current is flowing from 'b' to 'a') and Norton's current $I_N = -I_2 = \dfrac{24+20R}{15+8R}$

Norton‟s resistance R_N is computed by replacing all sources by their internal resistances while the short-circuit across the output terminal „a‟ and „b‟ is removed. From the circuit diagram fig. the Norton‟s resistance is obtained between the terminals „a‟ and „b‟.



$$R_N = (R \| 3) + 5 = \frac{3R}{3+R} + 5$$

Note that the maximum power will dissipate in load resistance when load resistance = Norton‟s resistance $R_N = R_L = 6.2\Omega$. To satisfy this condition the value of the resistance can be obtained from equation of R_N we get $R = 2\Omega$. The circuit now replaced by an equivalent Norton‟s current source.



Norton's equivalent circuit

The maximum power delivered by the given network to the load R_L=6.2Ω is thus given by

$$P_{max} = \frac{1}{4} \times I_N^2 R_L = \frac{1}{4} \times \left(\frac{24+20R}{15+8R} \right)^2 \times R_L = 6.61\, watts$$

Nodal Analysis in Electric Circuits

Definition of Nodal Analysis

Nodal analysis is a method that provides a general procedure for analyzing circuits using node voltages as the circuit variables. **Nodal Analysis** is also called the **Node-Voltage Method**.
In Node-Voltage Method, we can solve for unknown voltages in a circuit using KCL.

Some Features of Nodal Analysis:

- **Nodal Analysis** is based on the application of the Kirchhoff‟s Current Law (KCL).
- Having „n‟ nodes there will be „n-1‟ simultaneous equations to solve.
- Solving „n-1‟ equations all the nodes voltages can be obtained.
- The number of non reference nodes is equal to the number of Nodal equations that can be obtained.

Types of Nodes in Nodal Analysis

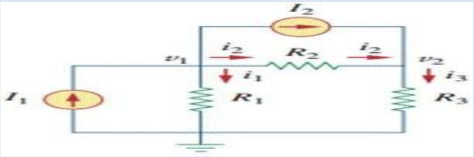
- Non Reference Node – It is a node which has a definite Node Voltage. e.g. Here Node 1 and Node 2 are the Non Reference nodes

- Reference Node – It is a node which acts a reference point to all the other node. It is also called the Datum Node.

Solving of Circuit Using Nodal Analysis

Basic Steps Used in Nodal Analysis:

1. Select a node as the reference node. Assign voltages V_1, V_2… V_{n-1} to the remaining nodes. The voltages are referenced with respect to the reference node.
2. Apply KCL to each of the non reference nodes.
3. Use Ohm"s law to express the branch currents in terms of node voltages.



4.After the application of Ohm"s Law get the „n-1" node equations in terms of node voltages and resistances.

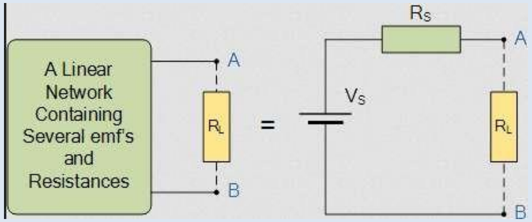
5.Solve „n-1" node equations for the values of node voltages and get the required node Voltages as result.

Thevenin's Theorem

Thevenin"s Theorem states that "Any linear circuit containing several voltages and resistances can be replaced by just one single voltage in series with a single resistance connected across the load". In other words, it is possible to simplify any electrical circuit, no matter how complex, to an equivalent two-terminal circuit with just a single constant voltage source in series with a resistance (or impedance) connected to a load as shown below.

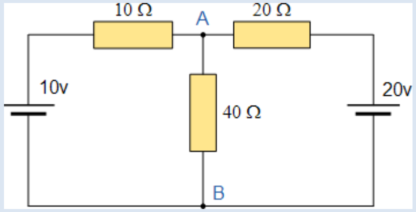
Thevenin's Theorem is especially useful in the circuit analysis of power or battery systems and other interconnected resistive circuits where it will have an effect on the adjoining part of the circuit.

Thevenin's equivalent circuit



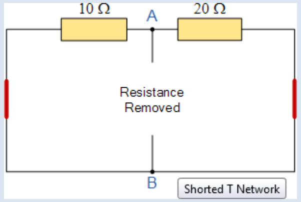
As far as the load resistor R_L is concerned, any complex "one-port" network consisting of multiple resistive circuit elements and energy sources can be replaced by one single equivalent resistance R_s and one single equivalent voltage V_s. R_s is the source resistance value looking back into the circuit and V_s is the open circuit voltage at the terminals.

<u>Find the load current in 40 Ω in given fig using Thevenin's theorem</u>



Firstly, to analyze the circuit we have to remove the centre 40Ω load resistor connected across the terminals A-B, and remove any internal resistance associated with the voltage source(s). This is done by shorting out all the voltage sources connected to the circuit, that is v = 0, or open circuit any connected current sources making i = 0. The reason for this is that we want to have an ideal voltage source or an ideal current source for the circuit analysis.

The value of the equivalent resistance, Rs is found by calculating the total resistance looking back from the terminals A and B with all the voltage sources shorted. We then get the following circuit.



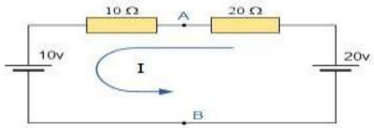
Find the Equivalent Resistance (Rs)

10Ω resistor in parallel with 20Ω resistor

R_s = 10*20/10+20= 200/30= 6.67Ω

The voltage Vs is defined as the total voltage across the terminals A and B when there is an open circuit between them. That is without the load resistor R_L connected.

Find the Equivalent Voltage (Vs)



We now need to reconnect the two voltages back into the circuit, and as $V_S = V_{AB}$ the current flowing around the loop is calculated as:

$$I = \frac{V}{R} = \frac{20v - 10v}{20\Omega + 10\Omega} = 0.33\ amps$$

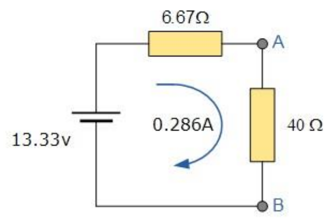
This current of 0.33 amperes (330mA) is common to both resistors so the voltage drop across the 20Ω resistor or the 10Ω resistor can be calculated as:

$$V_{AB} = 20 - (20\Omega \times 0.33amps) = 13.33\ volts.$$

or

$$V_{AB} = 10 + (10\Omega \times 0.33amps) = 13.33\ volts, the same.$$

Then the Thevenin's Equivalent circuit would consist or a series resistance of 6.67Ω and a voltage source of 13.33v. With the 40Ω resistor connected back into the circuit we get:



and from this the current flowing around the circuit is given as:

$$I = \frac{V}{R} = \frac{13.33v}{6.67\Omega + 40\Omega} = 0.286\ amps$$

What is the Maximum Power Transfer Theorem?

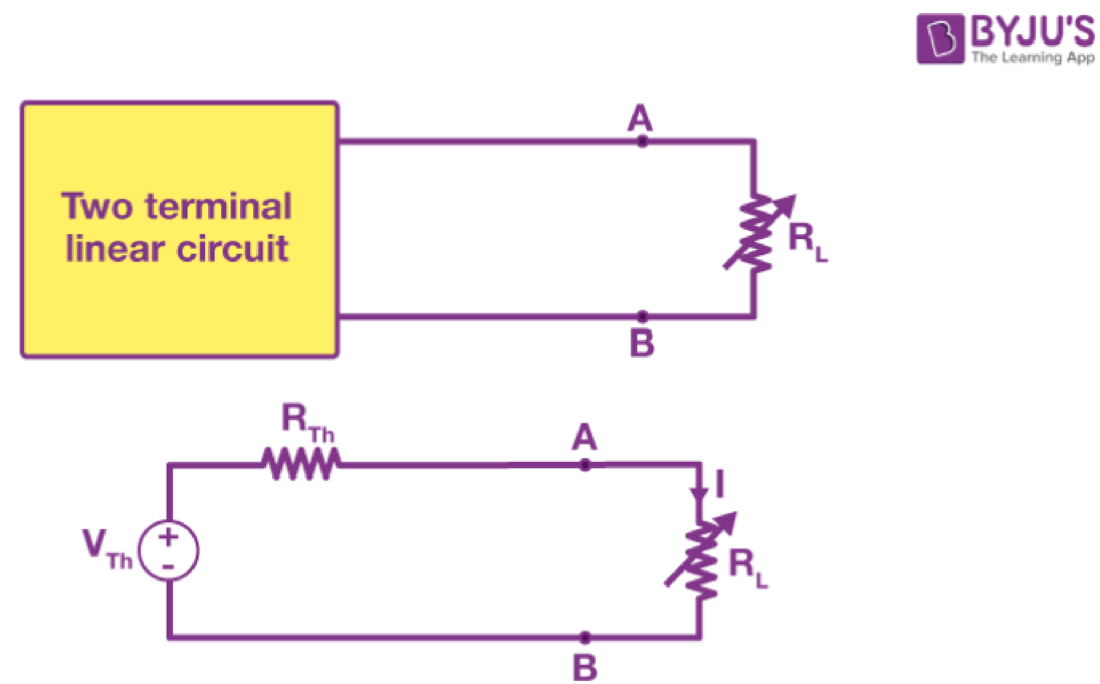
Maximum Power Transfer Theorem explains that to generate maximum external power through a finite internal resistance (DC network), the resistance of the given load must be equal to the resistance of the available source.

In other words, the resistance of the load must be the same as Thevenin's equivalent resistance.

In the case of AC voltage sources, maximum power is produced only if the load impedance's value is equivalent to the complex conjugate of the source impedance.

Maximum Power Transfer Formula

As shown in the figure, a dc source network is connected with variable resistance R_L.



Maximum Power Transfer Theorem Proof

The Maximum Power Transfer Theorem aims to figure out the value R_L, such that it consumes maximum power from the source.

$$I = \frac{V_{Th}}{R_{Th}+R_L}$$

The total power connected to the resistive load,

$$P_L = I^2 R_L = \left(\frac{V_{Th}}{R_{Th}+R_L}\right)^2 \times R_L$$

P_L can be maximized by adjusting R_L, therefore highest power can be generated when
$(dP_L/dR_L) = 0$

But,

$$\frac{dP_L}{dR_L} = \frac{1}{\left[(R_{Th}+R_L)^2\right]^2}\left[(R_{Th}+R_L)^2 \frac{d}{dR_L}\left(V_{Th}^2 R_L\right) - V_{Th}^2 R_L \frac{d}{dR_L}(R_{Th}+R_L)^2\right]$$

$$= \frac{1}{(R_{Th}+R_L)^4}\left[(R_{Th}+R_L)^2 V_{Th}^2 - V_{Th}^2 R_L \times 2(R_{Th}+R_L)\right]$$

$$= \frac{V_{Th}^2(R_{Th}+R_L-2R_L)}{(R_{Th}+R_L)^3} = \frac{V_{Th}^2(R_{Th}-R_L)}{(R_{Th}+R_L)}$$

$$\frac{dP_L}{dR_L} = 0$$

$$\frac{V_{Th}^2(R_{Th}-R_L)}{(R_{Th}+R_L)^3} = 0$$

$$(R_{Th} - R_L) = 0$$

$$R_{Th} = R_L$$

So, the highest power transmitted to the load resistance is,

$$P_{max} = \frac{V_{Th}^2}{4R_{Th}}$$

Application of Maximum Power Transfer Theorem

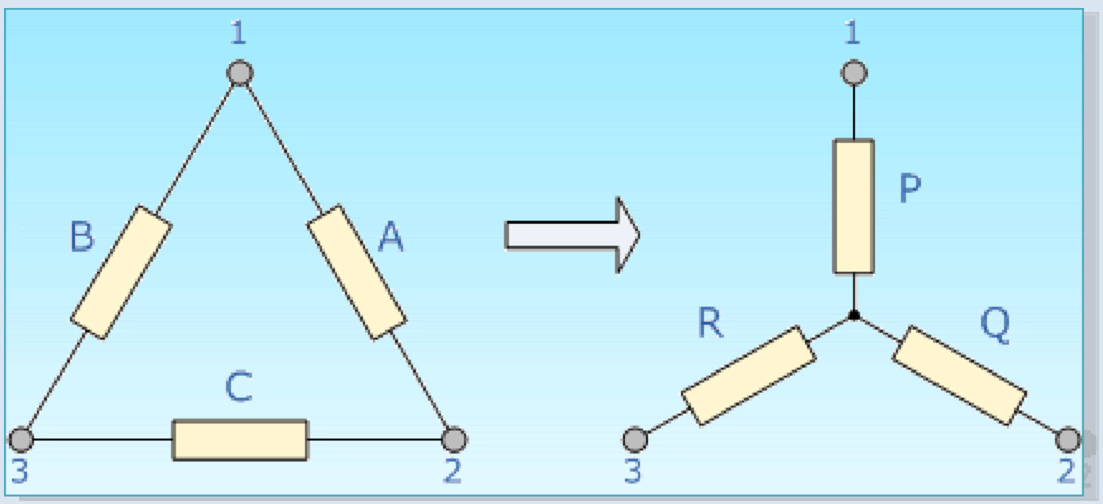
The presence of linked sources enables the network to be active, so the Maximum Power Transfer Theorem is applied for active networks & passive networks.

- Maximum Power Transfer Theorem can also be implemented to linear networks, the network system, along with R, L, C, & restrained linear sources as elements.
- Maximum power transfer theorem functions only when there is a variable load. If not, choose the least available internal sources of impedance, which paves in maximum current through the fixed load. Consequently, maximum power is expelled by the load circuit.
- Large sound systems are built around this process. Maximum power transfer is generated in the circuit by making the speaker's (load) resistance equivalent to the resistance of the amplifier. Once the speaker and amplifier have equal resistance, both are considered harmonised.
- Another application is on the relationship between the starter motor and the battery of a car engine. Power applied to the starter will rely on the effectual resistance of the motor and battery resistance. When their values are equal, the highest power will be transmitted to kickstart the engine.

EQUIVALENT THREE-TERMINAL NETWORKS

DELTA to WYE

- The equivalent resistance of each arm to the wye is given by the **PRODUCT** of the two delta sides that meet at its end divided by the **SUM** of the three delta resistances.



$$P = \frac{AB}{A+B+C}$$

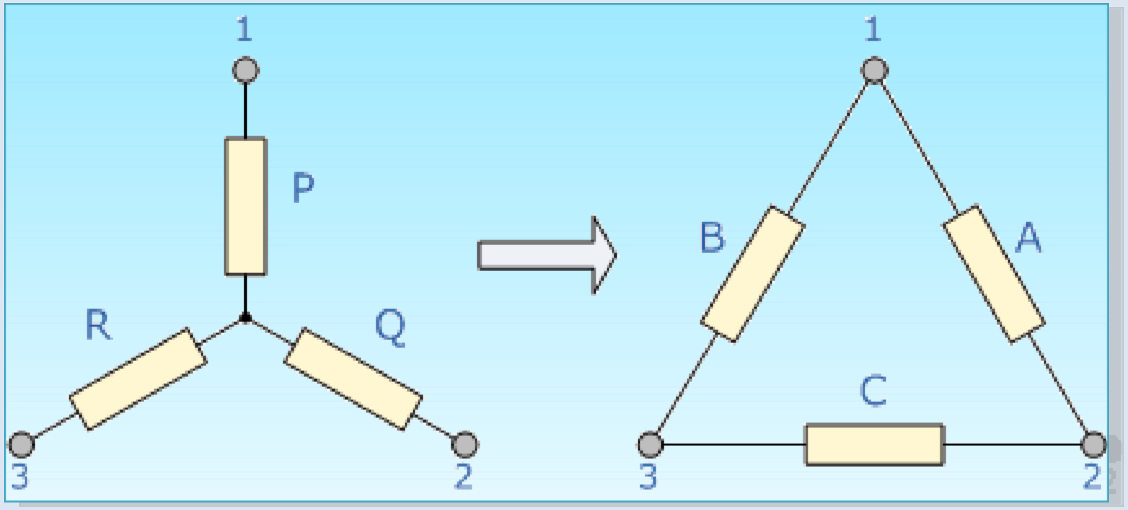
$$Q = \frac{AC}{A+B+C}$$

$$R = \frac{BC}{A+B+C}$$

EQUIVALENT THREE-TERMINAL NETWORKS

WYE to DELTA

- The equivalent delta resistance between any two twrminals is given by the **SUM** of a star resistance between those terminals PLUS the **PRODUCT** of these two star resistances DIVIDED by the third resistance.

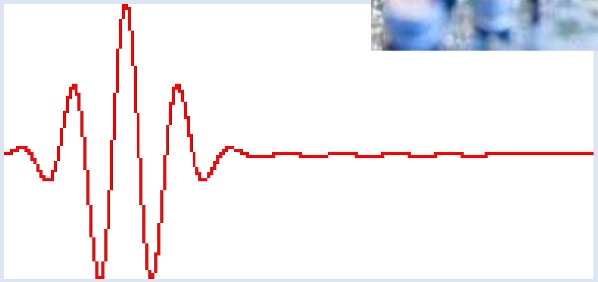


$$A = \frac{PQ + QR + RP}{R}$$

$$B = \frac{PQ + QR + RP}{Q}$$

$$C = \frac{PQ + QR + RP}{P}$$

Time-domain analysis of first-order RL and RC circuits,



Time-domain analysis of first-order RL and RC circuits

- Analysis of response of circuit consisting of R, L, C voltage source , current source & switches to **sudden** application of voltage or current is called as **Time domain Analysis & Transient Response**.
- When A.C. or D.C. voltage source is connected to circuit, a **steady current** can be calculated by many methods , already discussed . (Ohm's law).
- It is also assumed that circuit elements R, L, C are constant and source is very strong to absorb any disturbances.
- Amongst basic circuit elements Resistor is energy dissipating component & Inductor , Capacitor are energy storing elements. (electro magnetic & Electro static)
- Response of these elements to nature of source and disturbance varies from source to source.
- Transients (current or voltage lasting for short duration) in circuit is due to energy storing elements.
- For source free circuit transients response is called as **Natural Response** .
- For circuit with source transient response is called as **Forced Response**.

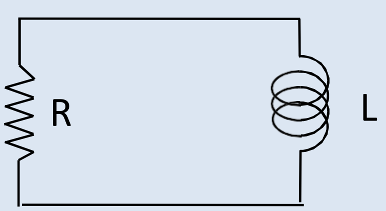
Time-domain analysis of first-order RL and RC circuits

- Disturbance in steady operation of circuit is unavoidable & can be of any type as below

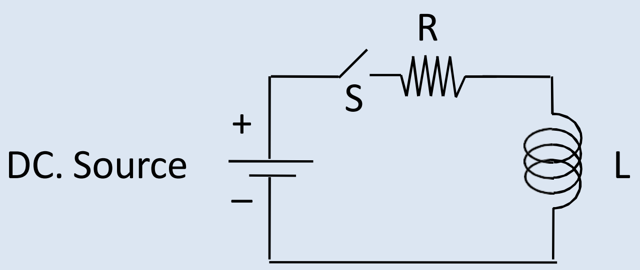
  I. Any circuit suddenly connected to source or disconnected from source.
  II. Sudden change in applied voltage from one level tom another
  III. Faults like short circuit or open circuit.

- After disturbance current or voltage shall have two components

  I. Final steady state component $(t \to \infty)$
  II. Transient component lasting for short duration that may settle down to zero or final value



Source Free RL Network



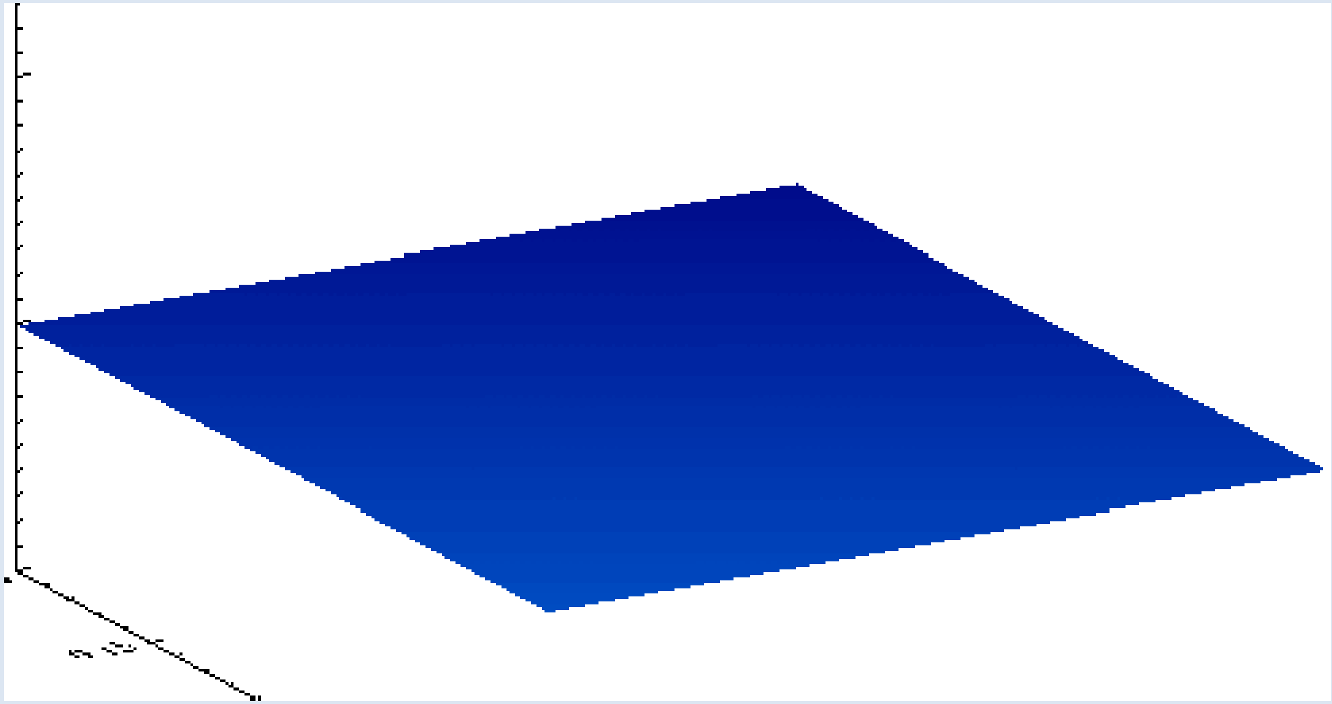
RL Network

Time-domain analysis of first-order RL and RC circuits

Steady System

Time-domain analysis of first-order RL and RC circuits

During Disturbance



Continuous Disturbance

Time-domain analysis of first-order RL and RC circuits

- Equations for these circuits, formed using KVL & KCL, consisting of basic elements contain derivatives & integrals of Currents / Voltages .

- Due to above facts equations are not algebraic but are differential in nature.

- Solutions of differential equations are functions of time & not constant as in case of purely resistive circuits.

Time-domain analysis of first-order RL and RC circuits

Series RL Circuit

Fig. 1 shows a series RL circuit connected across a DC source through a switch S. When switch 'S' is close at t>0 the as per KVL network equation will be …

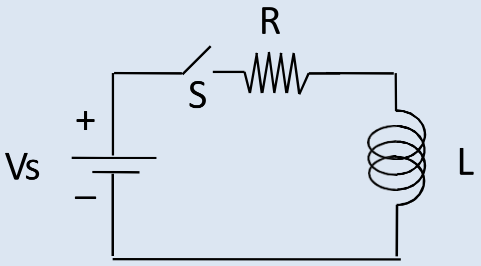


Fig. 1 Series RL Circuit

$$Ri(t) + L\frac{di(t)}{dt} = Vs$$ …………………. Eq. 1

Above equation is non homogenous equation linear differential equation of first order. The solution of Eq. 1 will give i(t) which consists of two components i.e.

i) Complimentary function ($i_n(t)$)

Which will satisfy $\frac{di(t)}{dt} + \frac{R}{L}i(t) = 0$

ii) Particular integral ($i_f(t)$)

Which will satisfy $Ri(t) + L\frac{di(t)}{dt} = Vs$

Thus complete solution may be written as …

$$i(t) = i_n(t) + i_J(t)$$ …………………. Eq. 2

Time-domain analysis of first-order RL and RC circuits

Series RL Circuit

$$i(t) = I_0 e^{-(\frac{R}{L})t} = I_0 e^{-(\frac{t}{\tau})}$$ .... Eq. 3

Where ζ = L/R time constant of RL circuit

Eq. 3 provides the natural reproduced and is reproduced below....

$$i_n(t) = Ke^{-(\frac{R}{L})t} = Ke^{-(t/\tau)}$$ .... Eq. 4

Eq. 1 can be written with (i(t) = I = constant)

$$RI + L\frac{dI}{dt} = Vs$$ .... Eq. 5

Since I = Constant

$$L\frac{dI}{dt} = 0$$

$$i_f(t) = I = \frac{Vs}{R}$$ .... Eq. 6

Substitute Eq. 4 & Eq.6 in Eq.2 yields the solution of Eq. 1

Time-domain analysis of first-order RL and RC circuits

Series RL Circuit

We get..

$$i(t) = Ke^{-t/\tau} + \frac{Vs}{R} = Ke^{-t/\tau} + I$$ ………………… Eq. 7

K is determined from initial condition i.e. t=0 Eq. 7 will be …

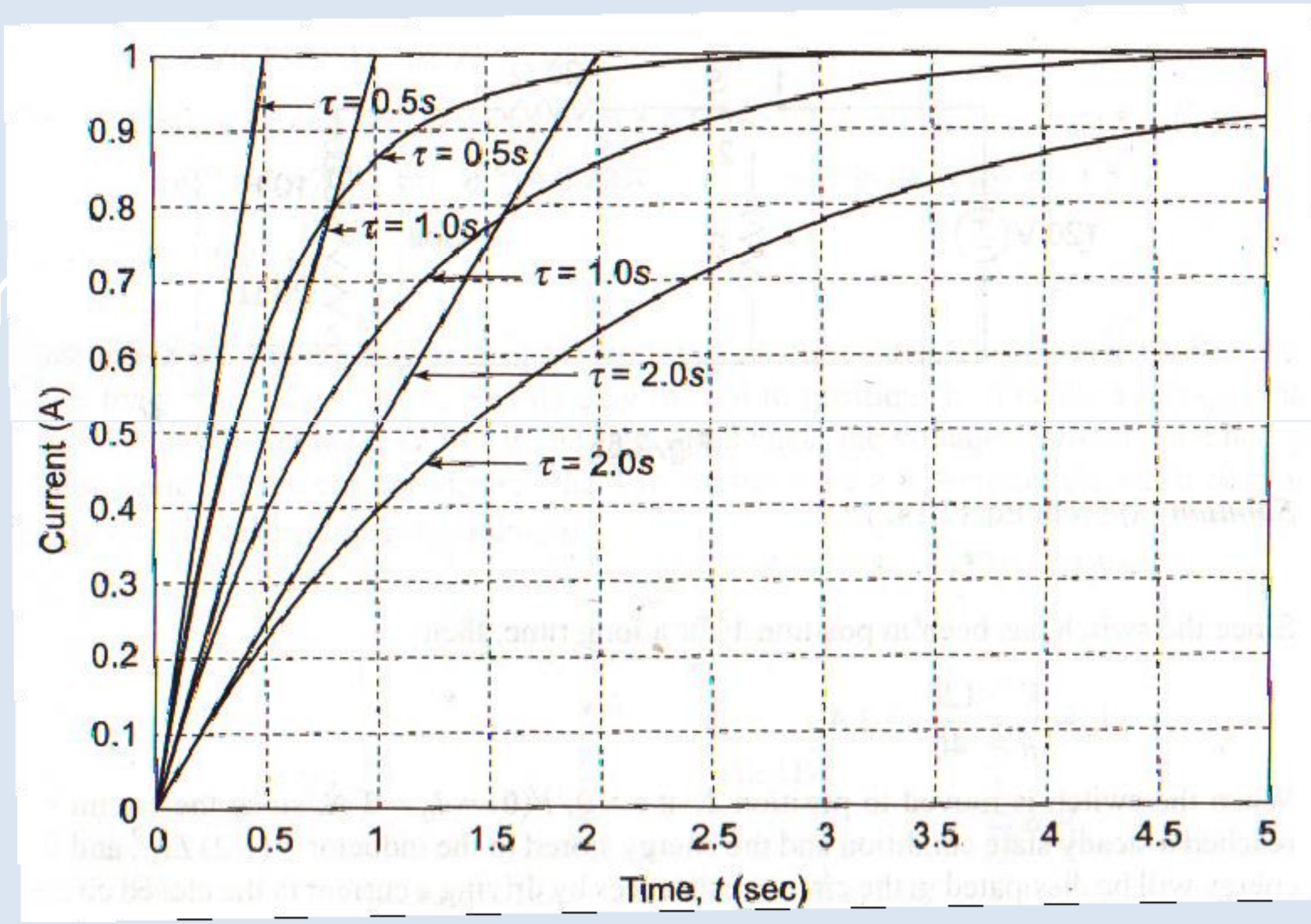
$$K = -\frac{Vs}{R} = -I$$ ………………… Eq. 8

Hence complete solution of Eq. 1 is given by

$$i(t) = \frac{Vs}{R}\left(1 - e^{-(\frac{R}{L}t)}\right)$$

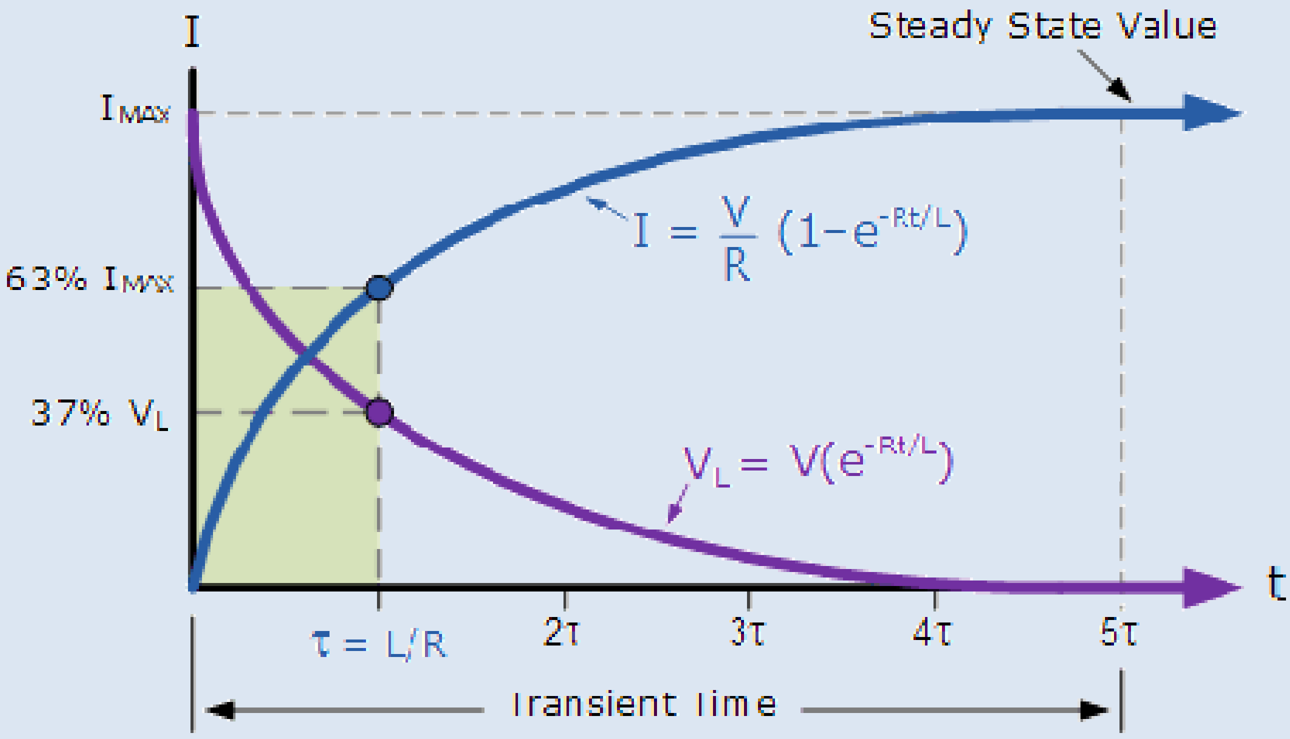
$$i(t) = I(1 - e^{-t/\tau})$$ For t >0

Series RL Circuit



Time-domain analysis of first-order RL and RC circuits

Series RL Circuit



Time-domain analysis of first-order RL and RC circuits

Series RC Circuit

Fig. 2 shows a series RC circuit connected across a DC source through a switch S. It is assumed that capacitor voltage id V_0 When switch 'S' is close at t>0 the as per KVL network equation will be …

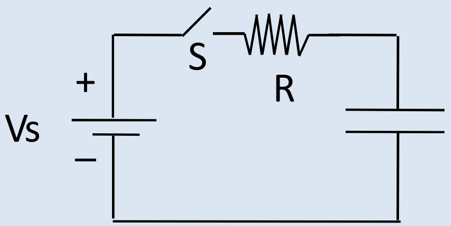


Fig. 2 Series RC Circuit

$$V_s - Ri(t) - v_C(t) = 0$$ …………………. Eq. 9

For analysis of circuit of Fig. 2 the capacitor voltage Vc(t) is chosen as variable.

Substituting $i(t) = c\dfrac{dv_c(t)}{dt}$ in Eq. 9 We get.

$$RC\frac{dv_c(t)}{dt} + v_c(t) = V_s$$ For t >0) …………………. Eq. 10

Above Eq.10 is like Eq. 1 it is also non homogenous equation linear differential equation of first order. Therefore solution, solution is also similar to Eq. 1. i.e.

$$v_c(t) = ke^{-t/\tau} + V_s$$ …………………. Eq. 11

Time-domain analysis of first-order RL and RC circuits

Series RC Circuit

In Eq. 11 the time constant is $\tau = RC$

By substituting initial condition Eq. 11 i.e. Vc = V_0 it leads to

$K = V_0 - V_s$

By substituting value of K in Eq. 11 and after simplification we get …

$$v_C(t) = V_0 e^{-\left(\frac{t}{\tau}\right)} + V_S\left(1 - e^{-\frac{t}{\tau}}\right)V$$ For t > 0 ………………….. Eq. 12

The expression for the current in the circuit is given by….

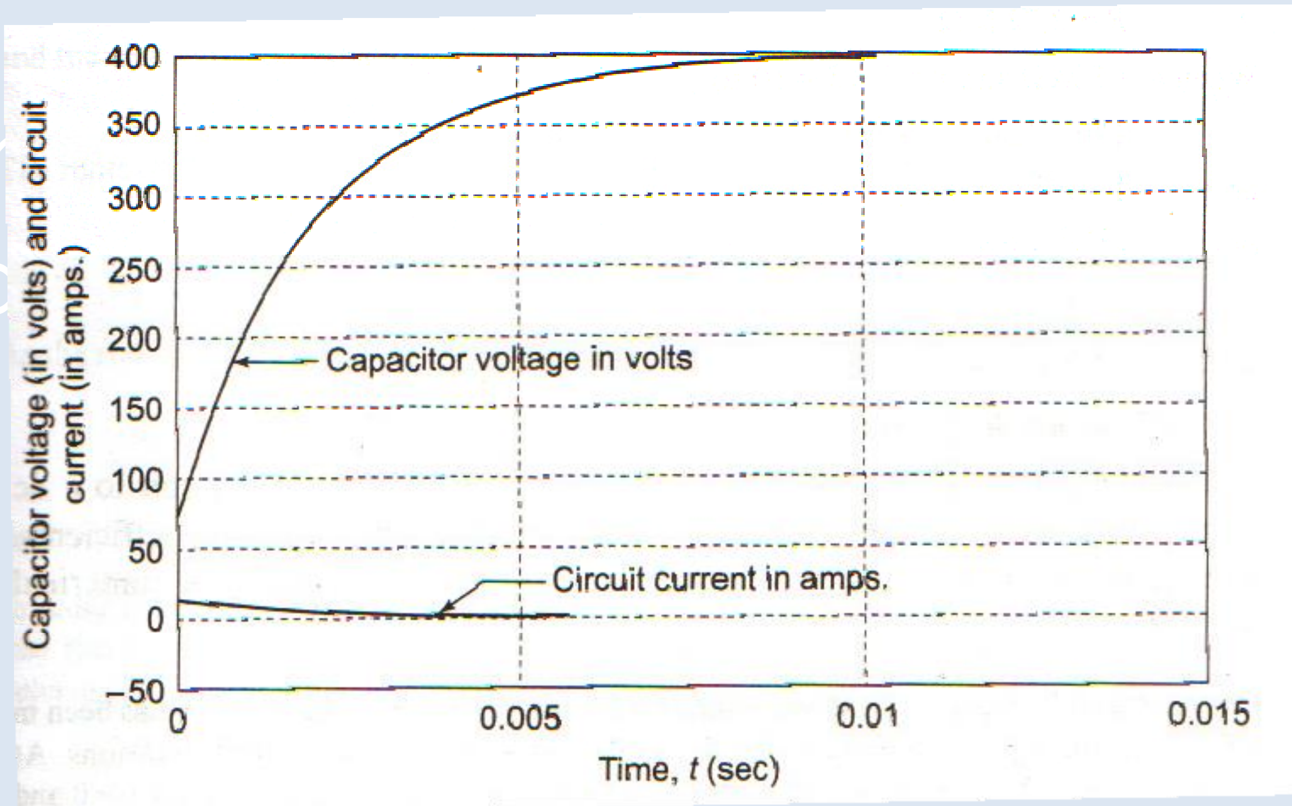
$$i(t) = C\frac{dv_C(t)}{dt}$$

$$i(t) = C\left[\left(-\frac{1}{\tau}V_0 e^{-\frac{t}{\tau}}\right) + \left(\frac{1}{\tau}V_S e^{-\frac{1}{\tau}}\right)\right]$$

$$i(t) = \frac{C}{RC}(V_S - V_0)e^{-\frac{t}{\tau}}$$

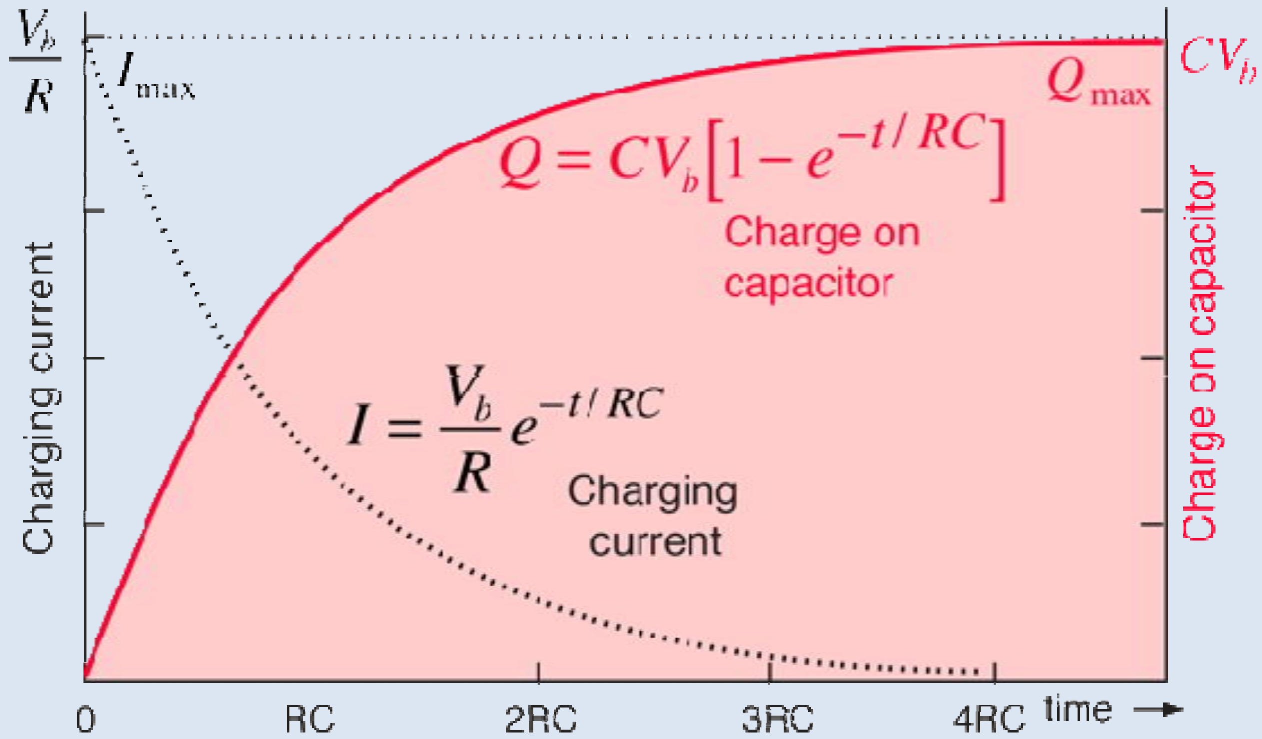
$$i(t) = \frac{(V_S - V_0)}{R}e^{-\frac{t}{\tau}}$$ ………………….. Eq. 13

Series RC Circuit



Time-domain analysis of first-order RL and RC circuits

Series RC Circuit



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